Optimal Linear Taxation of Positional Goods*

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Abstract

In this article I extend the Mirrlees (1971) framework to incorporate positional goods, that is, goods that are valued relative to other agents' consumption of the same good. Constrained efficient allocations can be implemented through a non-linear consumption tax on the positional good together with a marginal income tax with standard Mirrleesian properties, namely, no distortions at the extremes. The non-linearity in the former tax arises as I allow for individual specific positional externalities. Due to arbitrage opportunities that arise with the introduction of a non-linear consumption tax, I restrict this instrument to be *linear*. My numerical calculations indicate that the aggregate welfare losses of preventing arbitrage are very small, nevertheless, large distributional effects occur. For instance, when the positional externality is increasing in income, individuals at the high end of the income distribution experience large *gains* since for them, a flat tax effectively reduces the after tax price of positional goods. This generates a positive income effect that cannot be fully offset by increases in the marginal labor income tax as optimality requires no distortions at the top, thus higher consumption occurs. Conversely, individuals at the bottom of the distribution experience losses since they effectively face a higher after tax price of the positional good

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and consequently, a negative income effect. In this case, a marginal income tax reduction cannot offset such income effect due to incentive problems. Both effects are reduced when preferences over positional goods are non-homothetic as the income effect of price changes can be outweighed more effectively by adjustments in the marginal income tax.

1 Introduction

More that a hundred years ago Veblen (1899) coined the term conspicuous consumption to refer to the consumption incurred by individuals primarily with the goal of attaining status or social position. In modern capitalist societies at least some luxury goods posses this characteristic. Why is it that people are willing to pay fortunes to own a mansion in Beverly Hills, drive a brand new German convertible, have access to exclusive country clubs? No one can deny the intrinsic value derived from the consumption of those goods; however those purchases may be also motivated, at least partially, for positional considerations. In other words, for the status that such goods confers to the buyers.

By definition, status is a social ranking or standing. To the extend that agents preferences exhibit utility interdependence, the consumption of *positional* goods, such as luxuries, might impose a negative externality to the society. In other words, if agents preferences are sensitive to a ranking based on the consumption of a particular set of goods, the consumption of the "Joneses" may be harmful. Government intervention to correct this type of externalities may be desirable but for sure debatable. Some articles such as Frank (2005) have recently exposed arguments in favor of policy targeting this type of externalities. Among them, the author claims "tax cuts for the wealthy are spent largely on positional goods. Dollars that could have been used to pay for additional non positional goods have been spent instead on larger houses and more expensive cars".

The goal of this article is to conduct a formal normative analysis of taxation under the presence of consumption goods that generate positional externalities. I make my analysis in a framework similar to the one in Mirrlees (1971). Agents in my model are endowed with heterogeneous privately-known productivity. As is well known, these models capture a conflict between redistribution and incentives which results in an endogenous non-degenerate consumption distribution. As individuals preferences include relative consumption or positional considerations, consumption inequality becomes harmful and government intervention à la Pigou is desirable. In the analysis below, I parametrize the strength of positional considerations and analyze optimal tax policy.

Not surprisingly, constrained efficient allocations in this environment exhibit a wegde between non positional (for instance, necessities) and positional (for instance, luxuries) goods. The literature consensus however, is that taxing luxuries is not efficient. Using the Ramsey approach to optimal taxation, Atkinson and Stiglitz (1972) shows that it is optimal to tax goods with low income elasticities rather than high. Thus, necessities must be taxed higher than luxuries. A uniform commodity taxation result was obtained in Atkinson and Stiglitz (1976) under a framework with heterogeneous agents like the one analyzed in Mirrlees (1971). Remarkably, only the assumption of separability between consumption goods and leisure is needed to derive the latter result. Why is it that in this economic environment taxing positional goods such as luxuries is optimal? In the model that I present, taxation of the positional good occurs due to Pigouvian considerations. Thus, this instrument corrects over-consumption of a good that generates positional externalities. As preferences in this model display no utility interdependence, the uniform commodity taxation holds as in Atkinson and Stiglitz (1976) and all redistribution should be carried out through the labor income tax.

Under no restrictions on the class of taxes that can be used to implement optimal allocations, the presence of positional considerations implies that constrained efficient allocations can be implemented through a *non-linear* tax on the positional good in combination with a non-linear labor income tax with standard Mirrleesian properties, namely no distortions at the extremes. The non-linearity in the consumption of the positional good or "luxury tax" is driven by the fact that I allow agents to contribute to the positional externality in an arbitrary way. This is in line with the findings of Samano (2008) which finds that a progressive labor income tax may be partially rationalized as a Pigouvian one whose role is to correct consumption externalities. The estimations of the previous paper suggest that such externality may be increasing in income. The previous implementation however is subject to arbitrage opportunities across consumption goods since agents face a non-linear consumption tax on positional goods. Thus, a non-arbitrage constraint is imposed by equalizing the marginal rate of substitution between the positional and the non positional good across agents. I show that the resulting *double* constrained efficient allocations can be implemented through a *linear* positional tax together with a non-linear labor income tax. The non-linear income tax that implements double constrained efficient allocations differs from the one implementing constrained efficient ones as the former must offset income effects produced by the flattening in the "luxury tax".

Numerical calculations indicate that the aggregate welfare losses of preventing arbitrage are very small, nevertheless, large distributional effects occur. When the positional externality is increasing in income, individuals at the high end of the income distribution experience large *gains* since for them, a flat tax effectively reduces the after tax price of positional goods. This generates a positive income effect that cannot be fully offset by increases in the labor income tax as optimality requires no distortions at the top, thus the consumption of highly skilled individuals increases. Conversely, individuals at the bottom of the skills distribution experience losses since they effectively face a higher after tax price of the positional good and consequently, a negative income effect. In this case, a marginal income tax reduction cannot offset such income effect due to incentive problems. Both effects are reduced when preferences over positional goods are non-homothetic as price changes can be offset by small adjustments in the marginal income tax. My results suggest that the effectiveness of a linear "luxury tax" correcting positional externalities would crucially depend on the degree of non-homotheticity in preferences over positional and non positional goods.

The rest of the paper proceeds as follows. Section 2 presents the model and shows the characterization of constrained efficient allocations. Section 3 presents the characterization and one implementation of double constrained efficient allocations. Section 4 presents calculations of the endogenous distributions and optimal taxes for a parametrized version of the model. Finally, section 5 concludes.

2 The Model

Consider a static economy populated by a continuum of agents with heterogeneous productivity or skill. Let $\theta \in \Theta$, where $\Theta \equiv [\underline{\theta}, \bar{\theta}]$ and $0 < \underline{\theta} < \bar{\theta} < \infty$, be individual's productivity distributed according to the density $f : \Theta \to \mathbf{R}_{++}$. Productivity is *privately* known to each agent. An agent with productivity θ has a utility function of the form

$$U(c_n, c_l, y, C; \theta) = u(c_n, c_l) - \alpha C - v\left(\frac{y}{\theta}\right), \quad \alpha \in [0, 1)$$

where c_n is a necessity, c_l is a luxury good and y is effective output.[1] Moreover, let

$$C \equiv \int_{\Theta} [\omega c_n(\theta) + (1-\omega) c_l(\theta)] \psi(\theta) d\theta, \quad \omega \in [0, 1/2) \tag{1}$$

be society's endogenous consumption benchmark specified as a *weighted* average of necessities and luxuries. As usual, preferences satisfy $u_{c_n} > 0$, $u_{c_l} > 0$, $u(\cdot)$ is jointly strictly concave and

[1] As standard in this literature, I define effective labor as $y = \theta l$ where l is the amount of time worked.

$v(\cdot)$ is a convex function. Also, observe that according to the previous utility specification, $u_C = -\alpha$, thus, following the terminology of Dupor and Liu (2003), agents exhibit jealousy. Notice that with the assumption that $\omega < 1/2$, I capture the notion that luxuries provoke more jealousy than necessities as claimed by Frank (2008).[2] In other words, luxuries are more *positional* than necessities. Obviously, when $\omega = 0$, only luxuries are positional.

An *allocation* in this economy is $\{c_n(\theta), c_l(\theta), y(\theta)\}_{\theta\in\Theta}$, where $c_n : \Theta \to \mathbf{R}_+$, $c_l : \Theta \to \mathbf{R}_+$ and $y : \Theta \to \mathbf{R}_+$. Abstracting from government expenditure, I define an allocation $\{c_n(\theta), c_l(\theta), y(\theta)\}_{\theta\in\Theta}$ to be *feasible* if

$$\int_\Theta c_n(\theta) f(\theta) d\theta + \int_\Theta c_l(\theta) f(\theta) d\theta = \int_\Theta y(\theta) f(\theta) d\theta \tag{2}$$

Observe that in the previous definition I am assuming that both consumption goods are substitutes in production. This assumption is made for simplicity. A *reporting strategy* is a mapping $\sigma : \Theta \to \Theta$, where $\sigma(\theta)$ represents the skill announced by an agents with skill θ in a direct revelation game. Thus, making use of the Revelation Principle, an allocation is *incentive compatible* if

$$u(c_n(\theta), c_l(\theta)) - \alpha C - v\left(\frac{y(\theta)}{\theta}\right) \geq u(c_n(\sigma(\theta)), c_l(\sigma(\theta))) - \alpha C - v\left(\frac{y(\sigma(\theta))}{\theta}\right) \quad \forall \theta, \sigma(\theta) \in \Theta \tag{3}$$

Observe that since C cannot be affected unilaterally by a single agent, it is not a function of θ. An allocation that is incentive compatible and feasible is said to be *incentive-feasible*. Finally, let $g : \Theta \to \mathbf{R}_+$ be the density according to which individuals are weighted by the benevolent planner.

Definition 1. *A constrained efficient allocation is an allocation* $\{c_n^{sp}(\theta), c_l^{sp}(\theta), y^{sp}(\theta)\}_{\theta\in\Theta}$ *that maximizes the following planner problem*

$$\int_\Theta \left[u(c_n(\theta), c_l(\theta)) - \alpha C - v\left(\frac{y(\theta)}{\theta}\right)\right] g(\theta) d\theta \tag{4}$$

subject to $\{c_n(\theta), c_l(\theta), y(\theta)\}_{\theta\in\Theta}$ *being incentive-feasible and* $c_n(\theta), c_l(\theta), y(\theta) \geq 0 \forall \theta \in \Theta$.

[2] Empirical evidence of this fact is also presented in Carlsson, Johansson-Stenman, and Martinsson (2007) and Solnick and Hemenway (2005).

2.1 Characterization of Constrained Efficient Allocations

The following proposition states the necessary conditions that any *interior* constrained efficient allocation must satisfy. Let $\epsilon^{sp}(\theta) \equiv \frac{v'(\frac{y^{sp}(\theta)}{\theta})}{v''(\frac{y^{sp}(\theta)}{\theta})\frac{y^{sp}(\theta)}{\theta}}$.

Proposition 1. *Any interior constrained efficient allocation* $\{c_n^{sp}(\theta), c_l^{sp}(\theta), y^{sp}(\theta)\}_{\theta\in\Theta}$ *must be incentive-feasible and satisfy*

$$\frac{u_{c_n}(c_n^{sp}(\theta), c_l^{sp}(\theta))}{v'(\frac{y^{sp}(\theta)}{\theta})\frac{1}{\theta}} - 1 = \frac{\alpha}{\lambda}\frac{\omega\psi(\theta)}{f(\theta)} + \frac{u_{c_n}(c_n^{sp}(\theta), c_l^{sp}(\theta))}{\theta f(\theta)}\left[1 + \frac{1}{\epsilon^{sp}(\theta)}\right] I^{sp}(\theta) \quad \forall\theta\in\Theta \tag{5}$$

$$\frac{u_{c_l}(c_n^{sp}(\theta), c_l^{sp}(\theta))}{u_{c_n}(c_n^{sp}(\theta), c_l^{sp}(\theta))} = \frac{1 + \frac{\alpha(1-\omega)}{\lambda}\frac{\psi(\theta)}{f(\theta)}}{1 + \frac{\alpha\omega}{\lambda}\frac{\psi(\theta)}{f(\theta)}} \quad \forall\theta\in\Theta \tag{6}$$

where

$$\lambda = \frac{1 - \omega\alpha\int_\Theta \frac{\psi(\theta)}{u_{c_n}(c_n^{sp}(\theta), c_l^{sp}(\theta))}d\theta}{\int_\Theta \frac{f(\theta)}{u_{c_n}(c_n^{sp}(\theta), c_l^{sp}(\theta))}d\theta} \tag{7}$$

$$I^{sp}(\theta) \equiv \int_{\underline{\theta}}^{\theta}\left[\frac{g(t)}{\lambda} - \frac{f(t)}{u_{c_n}(c_n^{sp}(t), c_l^{sp}(t))} - \frac{\alpha}{\lambda}\frac{\omega\psi(t)}{u_{c_n}(c_n^{sp}(t), c_l^{sp}(t))}\right]dt \quad \forall\theta\in\Theta \tag{8}$$

Proof. See Appendix A. □

According to Proposition 1, the marginal rate of substitution (MRS) between the necessity and the luxury varies across agents if $\psi(\theta)$ is different to $f(\theta)$ as observed in expression 6. For the sake of exposition, consider the case where $\omega = 0$, that is, only the luxury good generates positional externalities. Moreover, assume that the ratio $\frac{\psi(\theta)}{f(\theta)}$ is strictly increasing, that is, the consumption of more affluent individuals becomes more harmful for the society as a whole. In that case, it is optimal to have an increasing MRS between the luxury and the necessity as income goes up. The previous argument breaks in two cases: either when $\psi(\theta)$ equals $f(\theta)$ and when $\alpha = 0$. In both cases, the MRS across consumption goods is constant across agents. In the second case, under no utility interdependence, the MRS across goods equals one (marginal rate of transformation given the assumed technology) so the uniform commodity taxation result holds.

From a theoretical point of view, the case where the MRS is non-constant across agents is more challenging to analyze. The reason is that if the planner cannot observe agents' consumption, individuals could meet in a re-trading markets after being assigned their consumption bundle and exchange consumption goods at a given price. In this re-trading market, they all would end up equalizing their MRS between the luxury and the necessity to an equilibrium relative price. To expose this better, suppose that the non-constant wegde were to be implemented by a non-linear tax on the consumption of the positional good or "luxury tax". Then, individuals could enter into "non-exclusive" arrangements to exchange luxuries for necessities until all agents equalized their MRS.[3] In order to take into account the previous fact, we need to refine the notion of constrained efficiency in this environment. Before formally stating this, we need a few definitions.

3 Double Constrained Efficient Allocations

I start by posing the agent's problem in the re-trading market contingent on having announced being of $\sigma(\theta)$ type.

3.1 Agents's Problem

Given allocations $\{c_n(\theta), c_l(\theta), y(\theta)\}_{\theta\in\Theta}$ and price q, an agent who decides to report $\sigma(\theta)$ attains utility

$$V(\{c_n(\theta), c_l(\theta), y(\theta)\}_{\theta\in\Theta}, q \mid \sigma(\theta)) = \max_{x_n(\sigma(\theta)), x_l(\sigma(\theta))} u(x_n(\sigma(\theta)), x_l(\sigma(\theta))) - \alpha C - v\left(\frac{y(\sigma(\theta))}{\theta}\right) \tag{9}$$

s.t.

$$x_n(\sigma(\theta)) + qx_l(\sigma(\theta)) \leq c_n(\sigma(\theta)) + qc_l(\sigma(\theta))$$

$$x_n(\sigma(\theta)), x_l(\sigma(\theta)) \geq 0$$

where $x_n((\sigma(\theta))$ and $x_l((\sigma(\theta))$ is the private consumption of the necessity and the luxury respectively and q is the relative price of the luxury good. Moreover, I define

$$V(\{c_n(\theta), c_l(\theta), y(\theta)\}_{\theta\in\Theta}, q) \equiv \max_{\sigma(\theta)\in\Theta} V(\{c_n(\theta), c_l(\theta), y(\theta)\}_{\theta\in\Theta}, q \mid \sigma(\theta)) \tag{10}$$

which represents the utility level attained by optimally announcing $\sigma(\theta)$, given $\{c_n(\theta), c_l(\theta), y(\theta)\}_{\theta\in\Theta}$ and q.

[3]The term "non-exclusivity" is used to emphasize that agents are not constrained to trade with one single partner.

3.2 Equilibrium in the Re-Trading Market

An equilibrium in the re-trading market is strategies $\{\sigma(\theta)\}_{\theta\in\Theta}$, allocations $\{x_n(\theta), x_l(\theta)\}_{\theta\in\Theta}$ and a price q such that
i) Taking as given $\{c_n(\theta), c_l(\theta), y(\theta)\}_{\theta\in\Theta}$ and q, agents solve (9) and (10),
ii) Re-trading market clears

$$\int_\Theta x_n(\sigma(\theta))f(\theta)d\theta = \int_\Theta c_n(\theta)f(\theta)d\theta$$

$$\int_\Theta x_l(\sigma(\theta))f(\theta)d\theta = \int_\Theta c_l(\theta)f(\theta)d\theta \tag{11}$$

Let $\hat{V}(\{c_n(\theta), c_l(\theta), y(\theta)\}_{\theta\in\Theta})$ equals $V(\{c_n(\theta), c_l(\theta), y(\theta)\}_{\theta\in\Theta}, \hat{q})$ where $\hat{q}$ is the equilibrium price. Given the previous definitions, we are in a position to define efficiency in this environment.

Definition 2. *A double constrained efficient allocation is an allocation* $\{c_n^*(\theta), c_l^*(\theta), y^*(\theta)\}_{\theta\in\Theta}$ *that maximizes the following planner problem*

$$\int_\Theta \left[u(c_n(\theta), c_l(\theta)) - \alpha C - v\left(\frac{y(\theta)}{\theta}\right)\right] g(\theta)d\theta \tag{12}$$

$$s.\ t.$$

$$u(c_n(\theta), c_l(\theta)) - \alpha C - v\left(\frac{y(\theta)}{\theta}\right) \geq \hat{V}(\{c_n(\theta), c_l(\theta), y(\theta)\}_{\theta\in\Theta}) \tag{13}$$

$$\int_\Theta c_n(\theta)f(\theta)d\theta + \int_\Theta c_l(\theta)f(\theta)d\theta = \int_\Theta y(\theta)f(\theta)d\theta \tag{14}$$

and $c_n(\theta), c_l(\theta), y(\theta) \geq 0 \quad \forall\theta \in \Theta$.

Notice that the previous notion of constrained efficiency takes explicitly into account the re-trading market for consumption goods across agents as a constraint. Lemma 1 establishes an equivalence statement of the problem stated in Definition 2.

Lemma 1. *A double constrained efficient allocation* $\{c_n^*(\theta), c_l^*(\theta), y^*(\theta)\}_{\theta\in\Theta}$ *together with the equilibrium relative price of luxuries q is a solution to the planner's problem*

$$\max_{c_n(\cdot), c_l(\cdot), y(\cdot), q} \int_\Theta \left[u(c_n(\theta), c_l(\theta)) - \alpha C - v\left(\frac{y(\theta)}{\theta}\right)\right] g(\theta)d\theta \tag{15}$$

$$s.\ t.$$

$$\frac{u_{c_l}(c_n(\theta), c_l(\theta))}{u_{c_n}(c_n(\theta), c_l(\theta))} = q \quad \forall \theta \in \Theta \tag{16}$$

$$u(c_n(\theta), c_l(\theta)) - \alpha C - v\left(\frac{y(\theta)}{\theta}\right) \geq V(\{c_n(\theta), c_l(\theta), y(\theta)\}_{\theta \in \Theta}, q \mid \sigma(\theta)) \quad \forall \sigma(\theta) \neq \theta \tag{17}$$

$$\int_\Theta c_n(\theta) f(\theta) d\theta + \int_\Theta c_l(\theta) f(\theta) d\theta = \int_\Theta y(\theta) f(\theta) d\theta \tag{18}$$

and $c_n(\theta), c_l(\theta), y(\theta) \geq 0 \quad \forall \theta \in \Theta$.

Proof. The proof follows closely da Costa (2009). Let q be the equilibrium relative price associated with the allocation $\{c_n(\theta), c_l(\theta), y(\theta)\}_{\theta \in \Theta}$ and assume that (13) is satisfied. If (16) is violated, there is an alternative consumption choice that increases the agents utility holding strategy $\sigma(\theta) = \theta$ fixed. Because q is an equilibrium price, this violates (13). To see that (17) holds observe that $\hat{V}(\{c_n(\theta), c_l(\theta), y(\theta)\}_{\theta \in \Theta})$ is simply (10) at equilibrium price. Hence, there is no strategy $\sigma(\theta)$ that yields higher utility that truth-telling.

Now assume (16) and (17) are satisfied. When the price is q, agents find in their best interest to reveal their types truthfully, according to (17). This means that there is no strategy $\sigma(\theta)$ combined with optimal re-trading that increases utility for the agents at that price. Equation (16) implies that no-trade is optimal for the agent at the very same price q. No-trade trivially satisfies (11) which guarantees that q is indeed an equilibrium price. Therefore, (17) implies (13). □

Lemma 1 states that if individual allocations are such that the MRS between consumption goods is *constant* across agents, in equilibrium, re-trading does not occur. Observe that the optimal MRS between luxuries and necessities is yet to be determined.

Theorem 1. *A double constrained efficient allocation* $\{c_n^*(\theta), c_l^*(\theta), y^*(\theta)\}_{\theta \in \Theta}$ *together with the equilibrium relative price of luxuries* q *solve the following planner problem*

$$\max_{c_n(\cdot), c_l(\cdot), y(\cdot), q} \int_\Theta \left[u(c_n(\theta), c_l(\theta)) - \alpha C - v\left(\frac{y(\theta)}{\theta}\right)\right] g(\theta) d\theta \tag{19}$$

s. t.

$$\frac{u_{c_l}(c_n(\theta), c_l(\theta))}{u_{c_n}(c_n(\theta), c_l(\theta))} = q \quad \forall \theta \in \Theta, \tag{20}$$

$$u(c_n(\theta), c_l(\theta)) - \alpha C - v\left(\frac{y(\theta)}{\theta}\right) \geq u(c_n(\sigma(\theta)), c_l(\sigma(\theta))) - \alpha C - v\left(\frac{y(\sigma(\theta))}{\theta}\right) \forall \theta, \sigma(\theta) \in \Theta \tag{21}$$

$$\int_{\Theta} c_n(\theta) f(\theta) d\theta + \int_{\Theta} c_l(\theta) f(\theta) d\theta = \int_{\Theta} y(\theta) f(\theta) d\theta \tag{22}$$

and $c_n(\theta), c_l(\theta), y(\theta) \geq 0 \quad \forall \theta \in \Theta$.

Proof. Suppose that a deviating agent announces $\sigma(\theta) = \theta'$. Hence, she receives $c_n(\theta'), c_l(\theta')$. Suppose she re-trades, thus $[x_n(\theta'), x_l(\theta')] \neq [c_n(\theta'), c_l(\theta')]$. However, truthful agent θ' and the deviating agent impersonating θ' who reported $\sigma(\theta) = \theta'$ have the same income in the re-trading market. By strict concavity of preferences it must be the case that $[x_n(\theta'), x_l(\theta')] = [c_n(\theta'), c_l(\theta')]$, otherwise there would exist another bundle strictly preferred to $[c_n(\theta'), c_l(\theta')]$, implying that the previous bundle is not optimal for truthful agent θ', thus we arrive to a contradiction. The previous implies that agents do not re-trade in equilibrium. Having established the previous fact, it follows that $V(\{c_n(\theta), c_l(\theta), y(\theta)\}, q \mid \sigma(\theta)) = u(c_n(\sigma(\theta)), c_l(\sigma(\theta))) - \alpha C - v\left(\frac{y(\sigma(\theta))}{\theta}\right)$ which together with (21) implies that (17) is satisfied. The other side of the proof is trivially satisfied as (17) considers the maximum of $\sigma(\theta)$. □

3.3 Characterization of Double Constrained Efficient Allocations

The following proposition states the necessary conditions that any *interior double* constrained efficient allocation must satisfy. Let $\epsilon^*(\theta) \equiv \frac{v'(\frac{y^*(\theta)}{\theta})}{v''(\frac{y^*(\theta)}{\theta})\frac{y^*(\theta)}{\theta}}$.

Proposition 2. *Any interior double constrained efficient allocation* $\{c_n^*(\theta), c_l^*(\theta), y^*(\theta)\}_{\theta\in\Theta}$ *must be incentive-feasible and satisfy*

$$\frac{u_{c_n}(c_n^*(\theta), c_l^*(\theta))}{v'(\frac{y^*(\theta)}{\theta})\frac{1}{\theta}} - 1 = \frac{\alpha}{\lambda}\frac{\omega\psi(\theta)}{f(\theta)} + \frac{u_{c_n}(c_n^*(\theta), c_l^*(\theta))}{\theta f(\theta)}\left[1 + \frac{1}{\epsilon^*(\theta)}\right] I^*(\theta) \quad \forall \theta \in \Theta \tag{23}$$

$$\frac{u_{c_l}(c_n^*(\theta), c_l^*(\theta))}{u_{c_n}(c_n^*(\theta), c_l^*(\theta))} = \frac{\int_\Theta \frac{f(\theta)}{B^*(\theta)} d\theta + \frac{\alpha(1-\omega)}{\lambda}\int_\Theta \frac{\psi(\theta)}{B^*(\theta)} d\theta}{\int_\Theta \frac{f(\theta)}{B^*(\theta)} d\theta + \frac{\alpha\omega}{\lambda}\int_\Theta \frac{\psi(\theta)}{B^*(\theta)} d\theta} \quad \forall \theta \in \Theta \tag{24}$$

where

$$\lambda = \frac{1 - \omega\alpha \int_\Theta \frac{\psi(\theta)}{u_{c_n}(c_n^*(\theta), c_l^*(\theta))} d\theta}{\int_\Theta \frac{f(\theta)}{u_{c_n}(c_n^*(\theta), c_l^*(\theta))} d\theta} \tag{25}$$

$$B^*(\theta) \equiv \frac{u_{c_n c_l}(c_n^*(\theta), c_l^*(\theta))}{u_{c_n}(c_n^*(\theta), c_l^*(\theta))} - \frac{u_{c_l c_l}(c_n^*(\theta), c_l^*(\theta))}{u_{c_l}(c_n^*(\theta), c_l^*(\theta))} \quad \forall \theta \in \Theta \tag{26}$$

$$I^*(\theta) \equiv \int_{\underline{\theta}}^{\theta} \left[\frac{g(t)}{\lambda} - \frac{f(t)}{u_{c_n}(c_n^*(t), c_l^*(t))} - \frac{\alpha}{\lambda}\frac{\omega\psi(t)}{u_{c_n}(c_n^*(t), c_l^*(t))}\right] dt \quad \forall \theta \in \Theta \tag{27}$$

Proof. See Appendix A. □

3.4 Implementation of Double Constrained Efficient Allocations

Agents in this economy trade effective labor for consumption of the necessity and luxury. There is a single firm that employs all agents. It produces one unit of output for every unit of effective labor, y. Necessities and luxuries are perfect substitutes in production. Every unit of effective labor receives a payment of w. Agent are also subject to an income tax schedule $T(y(\theta))$, assumed to be twice differentiable and to induce no bunching and a *linear* tax on the luxury good τ. Without loss of generality, there are not taxes on the consumption of the necessity c_n. An agent with effective labor y pays $T(y(\theta))$ of taxes.

Thus, taking as given $T(y(\theta))$, τ, C and the wage w, the problem solved by the agent with productivity $\theta, \forall\theta \in \Theta$ is

$$\max_{c_n(\theta), c_l(\theta), y(\theta)} u(c_n(\theta), c_l(\theta)) - \alpha C - v\left(\frac{y(\theta)}{\theta}\right) \tag{28}$$

s.t.

$$c_n(\theta) + (1+\tau)c_l(\theta) \leq wy(\theta) - T(y(\theta))$$

$$c_n(\theta), c_l(\theta), y(\theta) \geq 0$$

Definition 3. *Given a labor tax $T(y(\theta))$, luxury tax τ and C, an equilibrium in this economy is an allocation $\{c_n^{eq}(\theta), c_l^{eq}(\theta), y^{eq}(\theta)\}_{\theta\in\Theta}$ and wage w^{eq} such that*
i. $(c_n^{eq}(\theta), c_l^{eq}(\theta), y^{eq}(\theta))$ *solve (28)* $\forall\theta \in \Theta$
ii. $C = \int_\Theta [\omega c_n^{eq}(\theta) + (1-\omega)c_l^{eq}(\theta)]\psi(\theta)d\theta$
iii. $w^{eq} = 1$
iv. Government balances its budget

$$\int_\Theta [T(y^{eq}(\theta)) + \tau c_l^{eq}(\theta)]f(\theta)d\theta = 0$$

v. $\int_\Theta c_n^{eq}(\theta)f(\theta)d\theta + \int_\Theta c_l^{eq}(\theta)f(\theta)d\theta = \int_\Theta y^{eq}(\theta)f(\theta)d\theta$

An allocation $\{c_n(\theta), c_l(\theta), y(\theta)\}_{\theta\in\Theta}$ is *implementable* by the income tax $T(y(\theta))$ and the luxury tax τ if $\{c_n(\theta), c_l(\theta), y(\theta)\}_{\theta\in\Theta}$ and w are an equilibrium.

3.5 Characterization of Optimal Income Tax and Linear Luxury Tax

Define the following tax mechanism $T : y \to \mathbf{R}$,

$$T(y(\theta)) = \begin{cases} y^*(\theta) - c_n^*(\theta) - (1+\tau)c_l^*(\theta) & \text{if } y(\theta) = y^*(\theta) \\ y(\theta) & \text{otherwise.} \end{cases} \tag{29}$$

where

$$\tau = \frac{\frac{\alpha(1-2\omega)}{\lambda}\int_\Theta \frac{\psi(\theta)}{B^*(\theta)}d\theta}{\int_\Theta \frac{f(\theta)}{B^*(\theta)}d\theta + \frac{\alpha\omega}{\lambda}\int_\Theta \frac{\psi(\theta)}{B^*(\theta)}d\theta} \tag{30}$$

together with

$$\frac{T'(y^*(\theta))}{1-T'(y^*(\theta))} = \frac{\alpha}{\lambda}\frac{\omega\psi(\theta)}{f(\theta)} + \frac{u_{c_n}(c_n^*(\theta), c_l^*(\theta))}{\theta f(\theta)}\left[1 + \frac{1}{\epsilon^*(\theta)}\right] I^*(\theta) \tag{31}$$

if $y(\theta) = y^*(\theta)$.

Proposition 3. *Any optimal allocation $\{c_n^*(\theta), c_l^*(\theta), y^*(\theta)\}$ can be implemented by an income tax schedule $T(y(\theta))$ defined by (29) and (31) and a flat tax τ on the positional good satisfying (30).*

Proof. In Appendix A. □

Corollary 1 (Proposition 3)**.** *Suppose $\omega = 0$, $u(c_n, c_l) = \left[\eta c_n^{1-\frac{1}{\sigma}} + (1-\eta)c_l^{1-\frac{\rho}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$, $\sigma < \eta, \eta \leq 1$, then the optimal marginal income tax and luxury tax satisfy*

$$\frac{T'(y^*(\theta))}{1-T'(y^*(\theta))} = \frac{u_{c_n}(c_n^*(\theta), c_l^*(\theta))}{\theta f(\theta)}\left[1 + \frac{1}{\epsilon^*(\theta)}\right]\int_{\underline{\theta}}^{\theta}\left[\frac{g(t)}{\lambda} - \frac{f(t)}{u_{c_n}(c_n^*(\theta), c_l^*(\theta))}\right]dt \tag{32}$$

$$\tau = \frac{\alpha}{\lambda}\frac{\int_\Theta c_l^*(\theta)\psi(\theta)d\theta}{\int_\Theta c_l^*(\theta)f(\theta)d\theta} \tag{33}$$

where $\lambda = \frac{1}{\int_\Theta \frac{f(\theta)}{u_{c_n}(c_n^*(\theta), c_l^*(\theta))}d\theta}$ *and* $u_{c_n}(c_n^*(\theta), c_l^*(\theta)) \equiv \left[\eta c_n^*(\theta)^{1-\frac{1}{\sigma}} + (1-\eta)c_l^*(\theta)^{1-\frac{\rho}{\sigma}}\right]^{\frac{1}{\sigma-1}} \eta c_n^*(\theta)^{-\frac{1}{\sigma}}$.

It is important to highlight that results of Corollary 1 hold with or without homothetic preferences since the utility function (known as *generalized* elasticity of substitution and introduced in Pakos (2006)) becomes homothetic when $\rho = 1$.[4] In section 4, I compute constrained and double constrained efficient allocations and implementing taxes using the previous functional form. Corollaries 2 and 3 show expressions for optimal taxes under another classes of preferences standard in the literature.

Corollary 2 (Proposition 3)**.** *Suppose* $\omega = 0$, $u(c_n, c_l) = \frac{1}{1-\sigma}{c_n}^{1-\sigma} + \frac{1}{1-\rho}{c_l}^{1-\rho}$, *where* $\sigma, \rho > 0$ *then the optimal marginal income tax and luxury tax satisfy*

$$\frac{T'(y^*(\theta))}{1 - T'(y^*(\theta))} = \frac{c_n^*(\theta)^{-\sigma}}{\theta f(\theta)} \left[1 + \frac{1}{\epsilon^*(\theta)}\right] \int_{\underline{\theta}}^{\theta} \left[\frac{g(t)}{\lambda} - \frac{f(t)}{c_n^*(t)^{-\sigma}}\right] dt \tag{34}$$

$$\tau = \frac{\alpha}{\lambda} \frac{\int_\Theta c_l^*(\theta)\psi(\theta)d\theta}{\int_\Theta c_l^*(\theta)f(\theta)d\theta} \tag{35}$$

where $\lambda = \frac{1}{\int_\Theta c_n^*(\theta)^\sigma f(\theta)d\theta}$.

Corollary 3 (Proposition 3)**.** *Suppose* $\omega = 0$, $u(c_n, c_l) = \frac{1}{1-\sigma}{c_n}^{1-\sigma} - e^{-\rho c_l}$, *where* $\sigma, \rho > 0$ *then the optimal marginal income tax and luxury tax satisfy*

$$\frac{T'(y^*(\theta))}{1 - T'(y^*(\theta))} = \frac{c_n^*(\theta)^{-\sigma}}{\theta f(\theta)} \left[1 + \frac{1}{\epsilon^*(\theta)}\right] \int_{\underline{\theta}}^{\theta} \left[\frac{g(t)}{\lambda} - \frac{f(t)}{c_n^*(t)^{-\sigma}}\right] dt \tag{36}$$

$$\tau = \frac{\alpha}{\lambda} \tag{37}$$

where $\lambda = \frac{1}{\int_\Theta c_n^*(\theta)^\sigma f(\theta)d\theta}$.

4 Welfare Losses (and Gains!) Due to Linear Taxation on Positional Goods

In this section I compute the model constrained efficient and double constrained efficient allocations and evaluate the welfare in both environments. I also compute the taxes that

[4]To observe this, notice that

$$\frac{\partial c_l}{\partial y}\frac{y}{c_l} = \frac{\left[\frac{(\sigma-\rho)}{(\sigma-1)}\frac{(1-\eta)}{\eta}\right]^{-\sigma} c_l^\rho + qc_l}{\rho\left[\frac{(\sigma-\rho)}{(\sigma-1)}\frac{(1-\eta)}{\eta}\right]^{-\sigma} c_l^\rho + qc_l}$$

where q is the relative price of the luxury good in terms of the necessity. Thus, $\frac{\partial c_l}{\partial y}\frac{y}{c_l} = 1$ if $\rho = 1$ and $\frac{\partial c_l}{\partial y}\frac{y}{c_l} > 1$ if $\rho < 1$. In the latter case, these preferences properly represent the good c_l as a luxury.

implement those allocations. For this quantitative exercise, I assume that preferences are represented by $U(c_n, c_l, C, y; \theta) \equiv u(c_n, c_l) - \alpha C - \frac{1}{1+\phi}\left(\frac{y}{\theta}\right)^{1+\phi}$, $\phi > 0$, where

$$u(c_n, c_l) = \left[\eta c_n^{1-\frac{1}{\sigma}} + (1-\eta) c_l^{1-\frac{\rho}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

with $\rho \leq 1$ and $\sigma < 1$ with $\sigma < \rho$. For $\rho < 1$, this specification of preferences exhibit *non-homotheticity*. That is, as income of individuals goes up the share of disposable income spent in the luxury good increases.[5] As closely estimated by Pakos (2006), I set $\sigma = 0.5$ and vary the parameter that measures the ratio of income elasticities between necessities and luxuries, ρ, to take values in $\{0.80, 0.91, 1\}$.[6] Recall that when $\rho = 1$, preferences exhibit homotheticity. I set $\eta = 0.75$ and $\phi \in \{0.5, 1.5, 3\}$.[7] The latter implies an uncompensated elasticity of labor supply of 2, 1/2 and 1/3 respectively. I assume that the support of the distribution is $\Theta = [1, \infty)$ although for computational purposes, I use the domain $[1, 6]$. I consider two cases for the exogenous distributions of the model. In the first one, skills density $f(\theta)$ is distributed according to a truncated *exponential* distribution with parameter $\lambda_f = 1$ whereas $g(\theta)$ and $\psi(\theta)$ follow the same distribution with parameters $\lambda_g = 1$ and $\lambda_\psi = 1.25$ respectively.[8] Notice that I assume that the planner is utilitarian since $\lambda_g = \lambda_f$. In the second case, skills are distributed according to a Pareto distribution with parameter k_f, k_g and k_ψ respectively. I set $k_f = k_g = 1.8$ and $k_\psi = 1.08$. As before, the planner is utilitarian. These distributions are plotted in Figure 1.

I present my calculations for $\omega = 0$, that is, in this world, only luxuries generate positional externalities. I let the parameter $\alpha \in \{0.05, 0.15\}$.[9] Welfare losses are measured as in Golosov and Tsyvinski (2007). That is, let

$$U_\Theta^{sp} \equiv \int_\Theta \left[u(c_n^{sp}(\theta), c_l^{sp}(\theta)) - \alpha C^{sp} - \frac{1}{1+\phi}\left(\frac{y^{sp}(\theta)}{\theta}\right)^{1+\phi}\right] g(\theta) d\theta,$$

[5] Ait-Sahalia, Parker, and Yogo (2004) also model consumption as a composite of necessities and luxuries using non-homothetic preferences.

[6] The value of ρ is also in line with the work of Costa (2001) who estimates income elasticities for food and recreation in the U.S. up to 1994.

[7] In Appendix C I also present an exercise where $\phi = 0.2$ which implies a uncompensated labor supply elasticity of 5!

[8] Recall that the exponential distribution is $f(\theta) = \lambda e^{-\lambda\theta}, \theta \in [0, \infty)$. Thus, a lower tail truncated exponential distribution at $\theta = \underline{\theta}$ is $f^t(\theta) = \frac{1}{1-F(\underline{\theta})} f(\theta)$ where $F(\theta) = 1 - e^{-\lambda\theta}$.

[9] Samano (2008) obtains a value of α for aggregate consumption close to 0.15 using U.S. and U.K. income data. Dynan and Ravina (2007) obtain values close to 0.10.

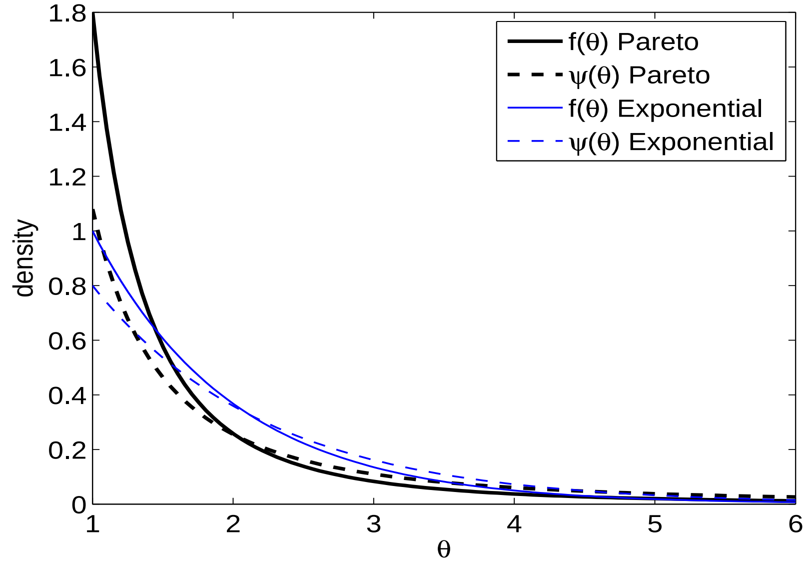

FIGURE 1: Skills and externality weighting distributions distributed Pareto and Exponential

thus I find the value λ_Θ such that

$$\int_\Theta \left[u((1+\lambda_\Theta)c_n^*(\theta), (1+\lambda_\Theta)c_l^*(\theta)) - \alpha C^* - \frac{1}{1+\phi}\left(\frac{y^*(\theta)}{\theta}\right)^{1+\phi} \right] g(\theta)d\theta = U_\Theta^{sp}.$$

In other words, I find the percentage increase in aggregate consumption λ_Θ that would deliver the same aggregate welfare in the double constrained efficient allocations than in the constrained efficient ones. The subindex Θ in the parameter λ makes explicit that welfare comparisons are made for *all* agents in the economy. To quantify *non-aggregate* welfare effects, I also calculate the value of $\lambda_L \equiv \lambda_{\{[\underline{\theta},\tilde{\theta}]|F(\tilde{\theta})=0.90\}}$ and $\lambda_H \equiv \lambda_{\{[\tilde{\theta},\bar{\theta}]|F(\tilde{\theta}=0.90)\}}$.[10]

Tables 1 and 2 show the optimal linear positional or "luxury tax" rate and welfare losses (and gains, whenever the variable λ is negative) for several parameters when skills are distributed according to an exponential distribution. Tables 3 and 4 show the same information when skills are Pareto distributed. As expected, the double constrained environment delivers lower aggregate welfare given that an extra constrained is imposed, namely, MRS between

[10]To be precise, λ_L is calculated as $\int_{\underline{\theta}}^{\tilde{\theta}} \left[u((1+\lambda_L)c_n^*(\theta), (1+\lambda_L)c_l^*(\theta)) - \alpha C^* - \frac{1}{1+\phi}\left(\frac{y^*(\theta)}{\theta}\right)^{1+\phi} \right] g(\theta)d\theta = U_L^{sp}$, where $U_L^{sp} \equiv \int_{\underline{\theta}}^{\tilde{\theta}} \left[u(c_n^{sp}(\theta), c_l^{sp}(\theta)) - \alpha C^{sp} - \frac{1}{1+\phi}\left(\frac{y^{sp}(\theta)}{\theta}\right)^{1+\phi} \right] g(\theta)d\theta$ and $F(\tilde{\theta}) = 0.90$. λ_H is calculated similarly.

goods is constant across agents.

My calculations indicate that the *aggregate* welfare losses suffered by taxing the positional good in a linear fashion are very low, particularly when the labor supply is reasonably inelastic. Thus a linear "luxury tax" does almost as well as a non-linear one in terms of aggregate welfare. Nevertheless, the previous policy has considerable distributional effects. Assuming the positional externality is increasing in income, individuals at the high end of the income distribution experience large *gains* under linear taxation of the positional good. The reason is the following: a flat luxury tax effectively reduces the price of luxuries for rich individuals. The drop in the price generates a positive income effect that cannot be offset by an increase in the marginal income tax as optimality requires no distortion at the top. Consequently, the consumption of agents at the top increases. When preferences are non-homothetic the gains of highly skilled individuals are reduced, not to negligible levels though, as the income effect generated by the drop in the price of luxuries is smaller. In this case, a small adjustment in the labor income tax is necessary to exert output from individuals at the high end of the income distribution. The opposite is true for individuals at the bottom of the skills distribution. They experience considerable welfare losses since a flat luxury tax increases the after tax price of luxuries. The previous generates a negative income effect that is offset by a reduction in the marginal income tax. Such reduction cannot fully offset the income effect since that would violate incentives. If preferences are non-homothetic, the adjustment in the income tax is smaller than in the homothetic case.

Observe that for $\alpha = 0.15$, the luxury tax is around 35% when the labor supply is inelastic ($\phi = 3, 1.5$) and reaches levels of around 40% when the labor supply is more elastic ($\phi = 0.50$). When positional considerations are weaker, $\alpha = 0.05$, this tax is around 11%. The more elastic the labor supply, the higher the optimal luxury tax. Also, notice that when the labor supply is inelastic, the *aggregate* welfare losses, represented by the parameter λ_Θ are no higher than 0.16%. When labor supply becomes more elastic, this loss increases up to 0.24%. Regarding *non-aggregate* welfare measures, when the labor supply is inelastic ($\phi = 3, 1.5$), individuals at the top decile of the skills distribution experience gains between 0.61% and 6.67%. Those gains, however, diminish when preferences are highly non-homothetic, the labor supply is elastic and positional concerns are weaker. Regarding welfare changes of individuals below the tenth decile of the skills distribution, the welfare losses due to linear taxation of positional goods are very high when preferences are homothetic, the labor supply is inelastic and the positional considerations are high. These losses considerably decrease when preferences exhibit non-homotheticity.

Figures 2 to 4 show comparative statics of the endogenous distributions of both consumption goods, effective output, utility levels and the optimal non-linear income tax and luxury tax that implement constrained efficient (CE) and double constrained efficient (DCE) allocations when skills are exponential distributed. Figures 5 to 7 show the same information when skills are distributed according to a Pareto distribution.[11] Figures 2 and 5 show the aforementioned distributions and taxes for different income elasticities, one of them corresponding to homothetic preferences. Observe that when preferences exhibit homotheticity, as the positional tax becomes flat, the marginal income tax drops to values very close to zero. The same is not true under non-homothetic preferences. In this case, the flattening of the positional tax only reduces the marginal labor income tax, however not to negligible levels. Figures 3 and 6 show CE and DCE allocations and taxes in both environments assuming two different labor supply elasticities and homothetic preferences in both cases. Clearly, allocations are very sensitive to this parameter. Notice, however, that even when the labor supply elasticity is low, the effective output distribution under the DCE is not too different to the CE one. The reason is that changes in the after tax price of positional goods are offset by reductions in the marginal labor income tax to the extend that incentives are not violated. Finally, Figures 4 and 7 present the same set of results considering a very elastic labor supply ($\phi = 0.50$) and non-homothetic preferences.[12] These figures confirm the direction of adjustments in the marginal income tax upon imposing linearity in the positional tax. The labor income tax goes down to offset income effects due to changes in the after tax price of positional goods. Nevertheless, since preferences are non-homothetic, changes in the labor tax are much more moderate than in the homothetic case. Appendix C presents additional comparative statics exercises.

Table 1: Summary of Variables when Distribution is Exponential, $\alpha = 0.15$

Variable		$\rho = 1$			$\rho = 0.91$			$\rho = 0.80$	
	$\phi = 3$	$\phi = 1.5$	$\phi = 0.50$	$\phi = 3$	$\phi = 1.5$	$\phi = 0.50$	$\phi = 3$	$\phi = 1.5$	$\phi = 0.50$
Linear Luxury Tax (τ)	34.58	35.94	41.35	35.77	36.87	41.08	37.24	37.91	40.03
Welfare Loss (λ_Θ)	0.06	0.06	0.13	0.05	0.08	0.17	0.06	0.06	0.10
Welfare Loss Bottom (λ_L)	0.94	0.89	1.49	0.71	0.69	0.92	0.52	0.45	0.43
Welfare Loss Top (λ_H)	-3.03	-2.23	-1.46	-2.62	-1.94	-1.17	-2.17	-1.58	-0.85

λ_f and λ_ψ imply that at the top of the distribution $\psi/f = 2.17$. All numbers are reported in percentage terms.

[11] Distributions were calculated using the Epanechnikov kernel with a bandwidth of $0.4 \times std(x) \times n^{-1/5}$ where x is the smoothed variable. The optimal bandwidth of Silverman (1986) over-smoothed the upper tail.

[12] For presentation purposes, I left out of the plot the bottom tail of the distributions in order to see clearly changes in the upper tail.

Table 2: Summary of Variables when Distribution is Exponential, $\alpha = 0.05$

Variable	$\rho = 1$			$\rho = 0.91$			$\rho = 0.80$		
	$\phi = 3$	$\phi = 1.5$	$\phi = 0.50$	$\phi = 3$	$\phi = 1.5$	$\phi = 0.50$	$\phi = 3$	$\phi = 1.5$	$\phi = 0.50$
Linear Luxury Tax (τ)	11.07	11.17	12.93	11.16	11.52	12.88	11.68	11.94	12.75
Welfare Loss (λ_Θ)	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.04
Welfare Loss Bottom (λ_L)	0.42	0.35	0.52	0.29	0.25	0.30	0.2	0.16	0.14
Welfare Loss Top (λ_H)	-1.34	-0.89	-0.51	-1.06	-0.73	-0.45	-0.8	-0.61	-0.23

λ_f and λ_ψ imply that at the top of the distribution $\psi/f = 2.17$. All numbers are reported in percentage terms.

Table 3: Summary of Variables when Distribution is Pareto, $\alpha = 0.15$

Variable	$\rho = 1$			$\rho = 0.91$			$\rho = 0.80$		
	$\phi = 3$	$\phi = 1.5$	$\phi = 0.50$	$\phi = 3$	$\phi = 1.5$	$\phi = 0.50$	$\phi = 3$	$\phi = 1.5$	$\phi = 0.50$
Linear Luxury Tax (τ)	32.93	35.6	43.87	33.51	35.29	41.96	34.21	35.52	39.32
Welfare Loss (λ_Θ)	0.14	0.16	0.21	0.15	0.15	0.24	0.12	0.12	0.14
Welfare Loss Bottom (λ_L)	1.79	1.79	3.15	1.33	1.23	1.85	0.88	0.86	0.67
Welfare Loss Top (λ_H)	-6.67	-4.94	-3.17	-5.71	-4.13	-2.78	-4.61	-3.73	-1.6

k_f and k_ψ imply that at the top of the distribution $\psi/f = 2.17$. All numbers are reported in percentage terms.

Table 4: Summary of Variables when Distribution is Pareto, $\alpha = 0.05$

Variable	$\rho = 1$			$\rho = 0.91$			$\rho = 0.80$		
	$\phi = 3$	$\phi = 1.5$	$\phi = 0.50$	$\phi = 3$	$\phi = 1.5$	$\phi = 0.50$	$\phi = 3$	$\phi = 1.5$	$\phi = 0.50$
Linear Luxury Tax (τ)	10.35	11.03	13.92	10.55	11.14	13.26	10.82	11.28	12.64
Welfare Loss (λ_Θ)	0.02	0.02	0.03	0.02	0.03	0.04	0.01	0.04	0.01
Welfare Loss Bottom (λ_L)	0.71	0.72	1.51	0.52	0.52	0.69	0.3	0.27	0.26
Welfare Loss Top (λ_H)	-2.68	-2.02	-1.43	-2.35	-1.78	-1.07	-1.75	-1.07	-0.72

k_f and k_ψ imply that at the top of the distribution $\psi/f = 2.17$. All numbers are reported in percentage terms.

5 Conclusions

In this paper I have introduced positional consumption goods within the Mirrlees (1971) framework. Positional goods are those whose valuation depends on an endogenous consumption benchmark. This consumption benchmark is a weighted average of all agents consumption of the positional good. As the contribution of individuals to the endogenous consumption benchmark differs from their population size, constrained efficient allocations exhibit a non-linear wegde between positional and no positional goods. Constrained efficient allocations can be implemented through a non-linear positional tax together with a

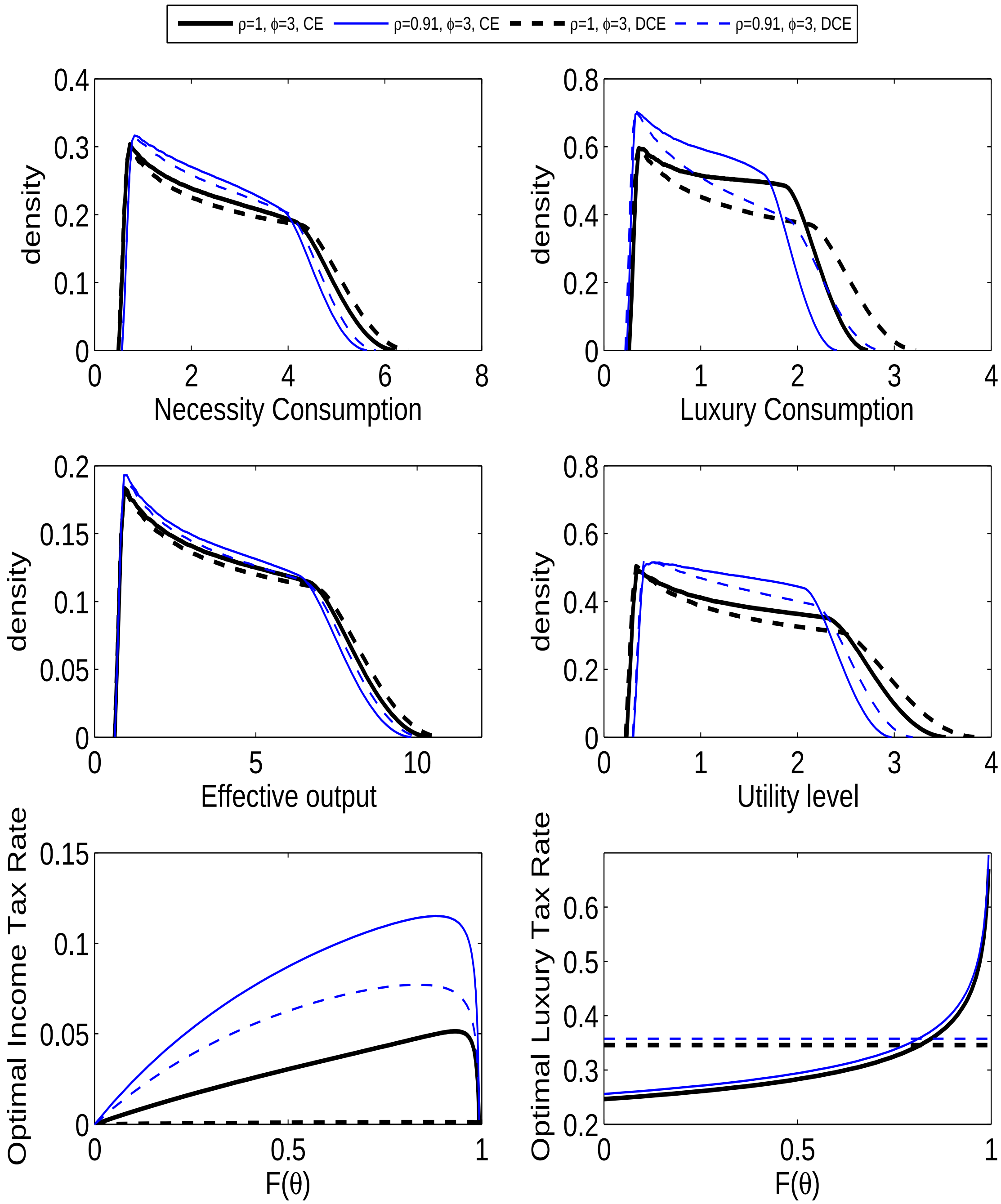

FIGURE 2: Endogenous distributions and optimal taxes when $\alpha = 0.15$ and distribution of skills is exponential. Changes in the income elasticity.

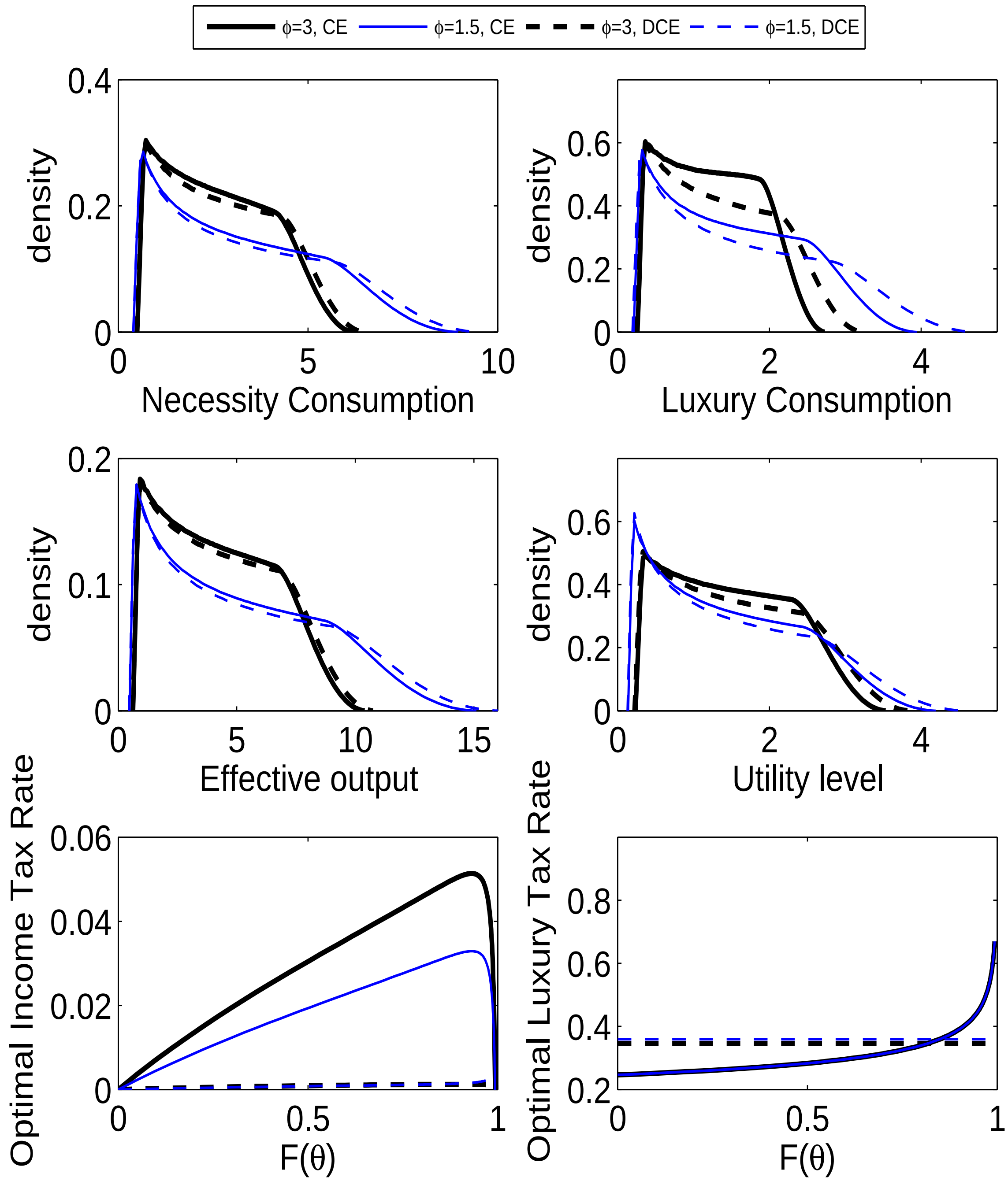

FIGURE 3: Endogenous distributions and optimal taxes when $\alpha = 0.15$, preferences are homothetic $\rho = 1$ and distribution of skills is exponential. Changes in the labor supply elasticity.

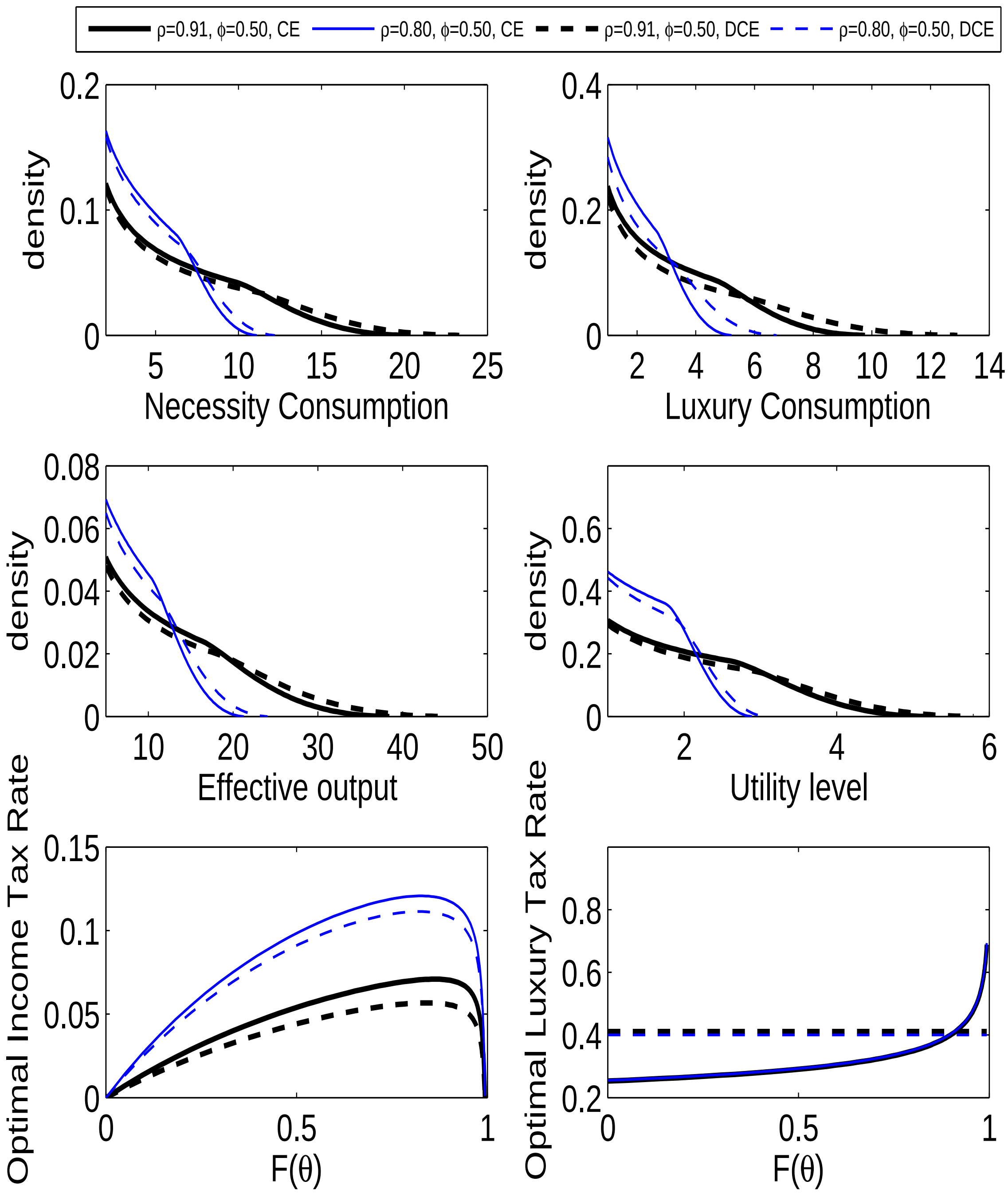

FIGURE 4: Upper tail of endogenous distributions and optimal taxes when $\alpha = 0.15$ and distribution of skills is exponential. Changes in the income elasticity.

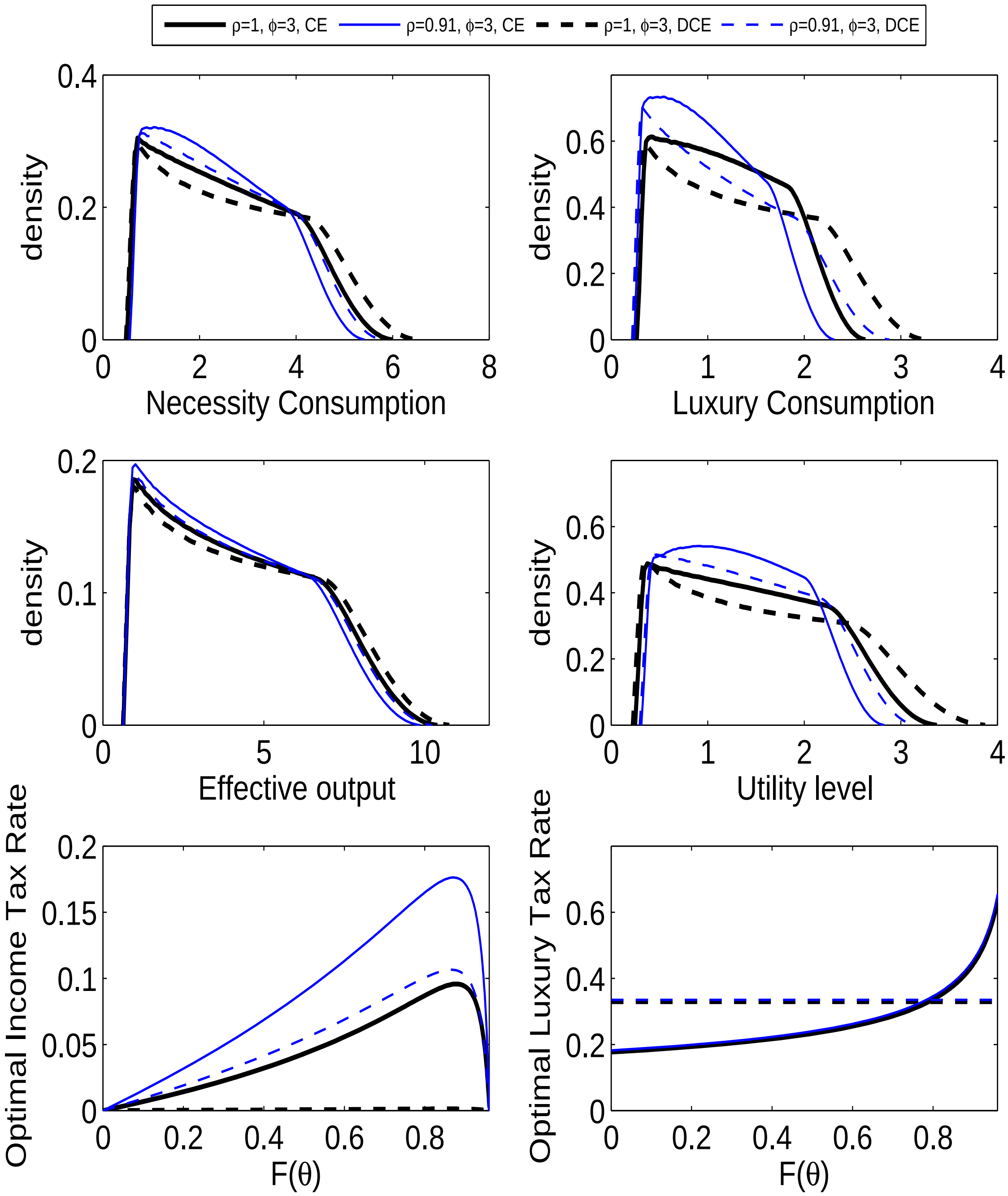

FIGURE 5: Endogenous distributions and optimal taxes when $\alpha = 0.15$ and distribution of skills is Pareto. Changes in the income elasticity.

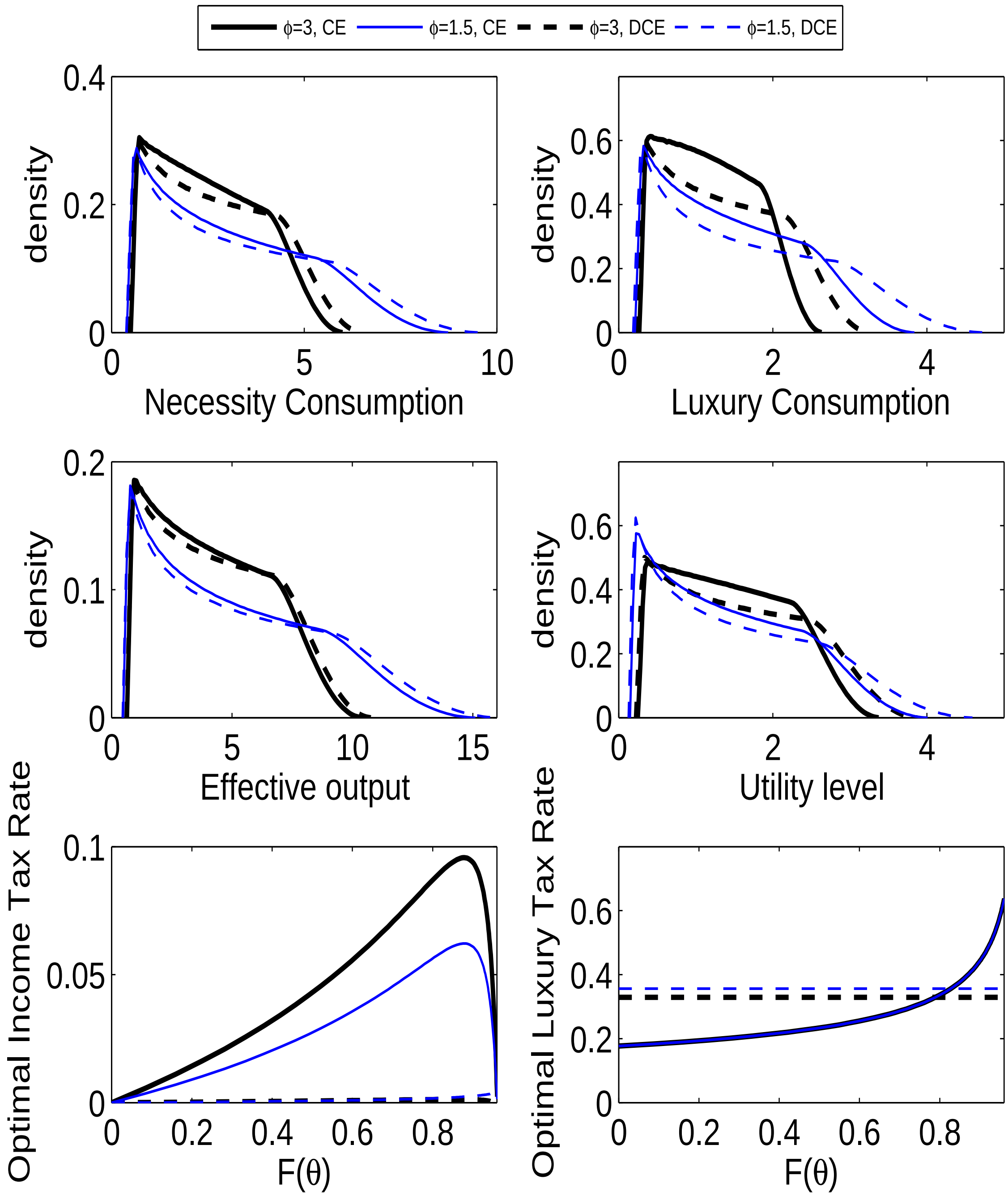

FIGURE 6: Endogenous distributions and optimal taxes when $\alpha = 0.15$, preferences are homothetic $\rho = 1$ and distribution of skills is Pareto. Changes in the labor supply elasticity.

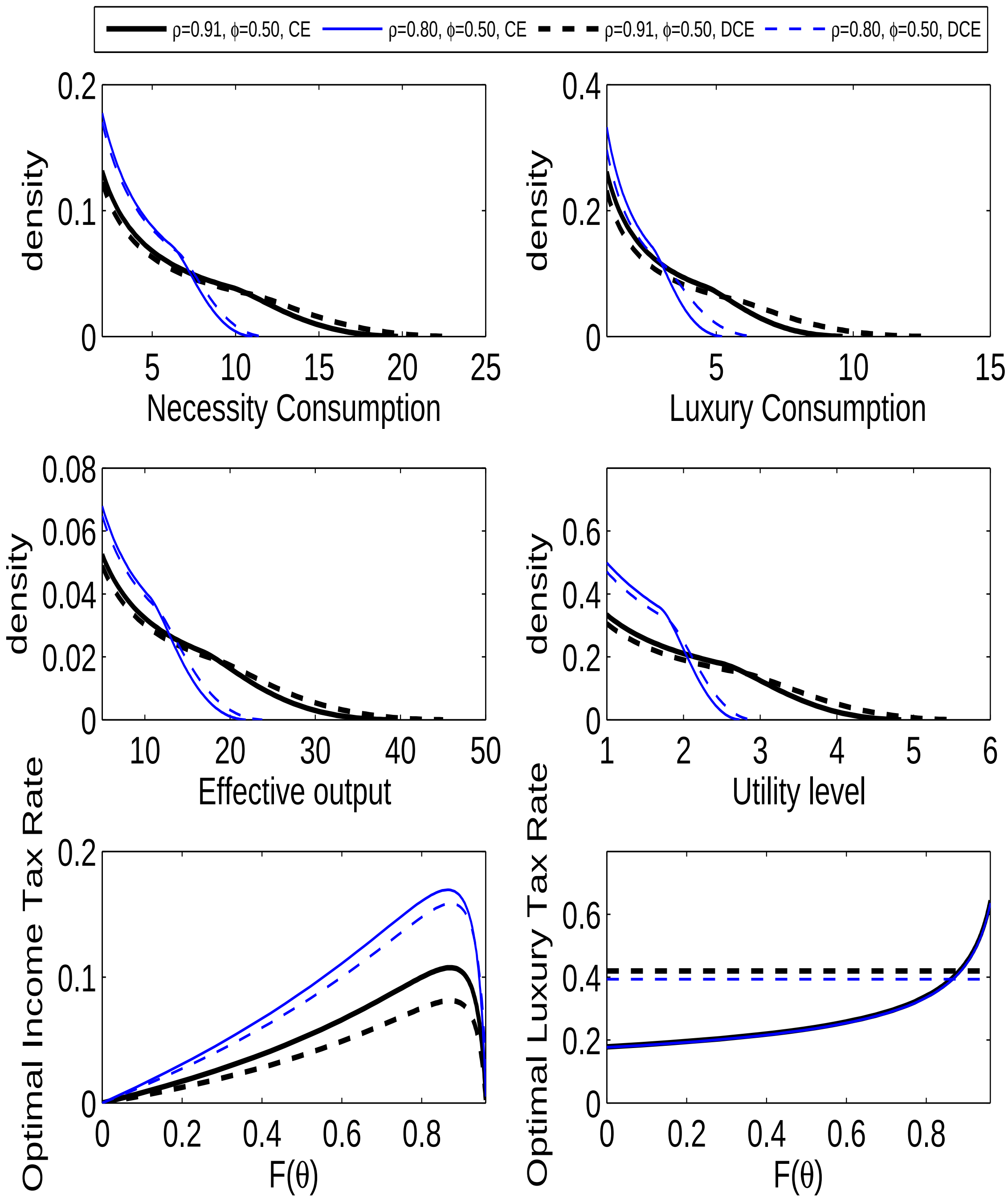

FIGURE 7: Upper tail of endogenous distributions and optimal taxes when $\alpha = 0.15$ and distribution of skills is Pareto. Changes in the income elasticity.

non-linear income tax with standard properties, namely, no distortions at the extremes. The previous implementation however, is subject to arbitrage opportunities across consumption goods. Thus, an extra constraint is imposed: the marginal rate of substitution between the positional and the positional good must be the same across agents. I have shown that the resulting *double* constrained efficient allocations can be implemented through a *linear* positional tax together with a non-linear labor income tax.

Aggregate welfare losses in the double constrained environment with respect to the constrained efficient environment are very low and in some cases negligible. Nevertheless, large distributional effects arise. Assuming that the positional externality is increasing in income, individuals at the high end of the income distribution experience large gains since for them, a flat tax effectively reduces the after tax price of positional goods, thus higher consumption occurs. This is so since the drop in the price generates a positive income effect that cannot be offset by an increase in the marginal income tax as optimality requires no distortion at the top. The opposite is true for individuals at the bottom of the skills distribution. They experience considerable welfare losses since a flat luxury tax increases the after tax price of luxuries. The previous generates a negative income effect that is offset by a reduction in the marginal income tax. Such reduction cannot fully offset the income effect since that would violate incentives. When preferences are non-homothetic, small adjustments in the income tax are more effective offsetting income effects derived from changes in the "luxury tax". My results suggest that the effectiveness of a linear consumption tax correcting positional externalities would crucially depend on the degree of non-homotheticity in preferences over positional and non positional goods.

An important extension of the current model is to incorporate a production technology whose marginal rate of transformation between luxuries and necessities is not constant. Such specification would allow us to incorporate in our quantitative analysis potential sharper changes in the output of the economy as a result of a good specific tax. Also, it is important to remark that the results presented assume that agents cannot buy positional goods in markets with different tax regimes. This consideration would impose an upper bound on this tax that is not considered in this model.

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Appendix

A Proofs

A.1 Proof of Proposition 1

Proof. Replicate proof of Proposition 2 setting $\eta(\theta) = 0 \quad \forall \theta \in \Theta$. □

A.2 Proof of Proposition 2

Proof. The first step is to transform the continuum of incentive compatibility constraints (3) into a first order condition. Let $\sigma(\theta) = \theta'$ and

$$W(\theta, \theta') \equiv u(c_n(\theta'), c_l(\theta')) - \alpha C - v\left(\frac{y(\theta')}{\theta}\right) \tag{38}$$

A necessary condition for truthful revelation of type is $\frac{\partial W(\theta,\theta')}{\partial \theta'}|_{\theta'=\theta} = 0$, therefore it follows that

$$u_{c_n}(c_n(\theta), c_l(\theta))\frac{\partial c_n(\theta)}{\partial \theta} + u_{c_l}(c_l(\theta), c_l(\theta))\frac{\partial c_l(\theta)}{\partial \theta} = v'\left(\frac{y(\theta)}{\theta}\right)\frac{y'(\theta)}{\theta} \quad \forall \theta \in \Theta \tag{39}$$

Moreover, under truthful revelation $W(\theta) = u(c_n(\theta), c_l(\theta)) - \alpha C - v\left(\frac{y(\theta)}{\theta}\right)$ and hence, $W'(\theta) = u_{c_n}(c_n(\theta), c_l(\theta))\frac{\partial c_n(\theta)}{\partial \theta} + u_{c_l}(c_n(\theta), c_l(\theta))\frac{\partial c_l(\theta)}{\partial \theta} - v'\left(\frac{y(\theta)}{\theta}\right)\frac{y'(\theta)}{\theta} + v'\left(\frac{y(\theta)}{\theta}\right)\frac{y(\theta)}{\theta^2}$, which together with (39) becomes

$$W'(\theta) = v'\left(\frac{y(\theta)}{\theta}\right)\frac{y(\theta)}{\theta^2} \quad \forall \theta \in \Theta. \tag{40}$$

Define the expenditure function $e(W(\theta), c_l(\theta), y(\theta), C; \theta)$ to satisfy $W(\theta) = u(e, c_l) - \alpha C - v\left(\frac{y(\theta)}{\theta}\right)$. Thus, the planner problem can be restated as

$$\max_{W(\cdot), c_l(\theta), y(\cdot), C, \kappa} \int_{\Theta} W(\theta) g(\theta) d\theta \tag{41}$$

s.t

$$\int_\Theta c_l(\theta)f(\theta)d\theta + \int_\Theta e(W(\theta), c_l(\theta), y(\theta), C; \theta)f(\theta)d\theta = \int_\Theta y(\theta)f(\theta)d\theta \tag{42}$$

$$W'(\theta) = v'\left(\frac{y(\theta)}{\theta}\right)\frac{y(\theta)}{\theta^2} \quad \forall\theta \in \Theta \tag{43}$$

$$e_{c_l}(W(\theta), c_l(\theta), y(\theta), C; \theta) = \kappa \quad \forall\theta \in \Theta \tag{44}$$

$$C \equiv \int_\Theta \left[\omega e(W(\theta), c_l(\theta), y(\theta), C; \theta) + (1-\omega)c_l(\theta)\right]\psi(\theta)d\theta \tag{45}$$

The corresponding Lagrangian is

$$\mathfrak{L}(W(\theta), c_l(\theta), y(\theta), C, \kappa, \lambda, \mu(\theta), \gamma, \eta(\theta)) = \int_\Theta W(\theta)g(\theta)d\theta$$

$$-\lambda\int_\Theta \left[c_l(\theta) + e(W(\theta), c_l(\theta), y(\theta), C; \theta) - y(\theta)\right]f(\theta)d\theta + \int_\Theta \mu(\theta)\left[W'(\theta) - v'\left(\frac{y(\theta)}{\theta}\right)\frac{y(\theta)}{\theta^2}\right]d\theta$$

$$+\gamma\left[C - \int_\Theta [\omega e(W(\theta), c_l(\theta), y(\theta), C; \theta) + (1-\omega)c_l(\theta)]\psi(\theta)d\theta\right]$$

$$+\int_\Theta \eta(\theta)\left[e_{c_l}(W(\theta), c_l(\theta), y(\theta), C; \theta) - \kappa\right]d\theta \tag{46}$$

Using integration by parts, it follows that

$$\int_\Theta \mu(\theta)W'(\theta)d\theta = \mu(\bar{\theta})W(\bar{\theta}) - \mu(\underline{\theta})W(\underline{\theta}) - \int_\Theta \mu'(\theta)W(\theta)d\theta \tag{47}$$

thus, we can reexpress the above Lagrangian as

$$\mathfrak{L}(W(\theta), c_l(\theta), y(\theta), C, \kappa, \lambda, \mu(\theta), \gamma, \eta(\theta)) = \int_\Theta W(\theta)g(\theta)d\theta$$

$$-\lambda\int_\Theta [c_l(\theta) + e(W(\theta), c_l(\theta), y(\theta), C; \theta) - y(\theta)]f(\theta)d\theta + \mu(\bar{\theta})W(\bar{\theta}) - \mu(\underline{\theta})W(\underline{\theta})$$

$$-\int_\Theta \mu'(\theta)W(\theta)d\theta - \int_\Theta \mu(\theta)v'\left(\frac{y(\theta)}{\theta}\right)\frac{y(\theta)}{\theta^2}d\theta$$

$$+\gamma\left[C-\int_\Theta[\omega e(W(\theta),c_l(\theta),y(\theta),C;\theta)+(1-\omega)c_l(\theta)]\psi(\theta)d\theta\right]$$

$$+\int_\Theta\eta(\theta)\left[e_{c_l}(W(\theta),c_l(\theta),y(\theta),C;\theta)-\kappa\right]d\theta \tag{48}$$

Assuming interior solution, it follows from first order conditions that

$W(\theta)$:

$$g(\theta)-\lambda f(\theta)e_W(W(\theta),c(\theta)_l,y(\theta),C;\theta)-\mu'(\theta)-\gamma\omega\psi(\theta)e_W(W(\theta),c_l(\theta),y(\theta),C;\theta)=0 \tag{49}$$

$y(\theta)$:

$$-\lambda e_y(W(\theta),c_l(\theta),y(\theta),C;\theta)f(\theta)+\lambda f(\theta)-\frac{\mu(\theta)}{\theta^2}v'\left(\frac{y(\theta)}{\theta}\right)\left[1+\frac{1}{\epsilon(\theta)}\right]$$

$$-\gamma\omega e_y(W(\theta),c_l(\theta),y(\theta),C;\theta)\psi(\theta)=0 \tag{50}$$

$c_l(\theta)$:

$$-\lambda e_{c_l}(W(\theta),c_l(\theta),y(\theta),C;\theta)f(\theta)-\lambda f(\theta)-\gamma\omega e_{c_l}(W(\theta),c_l(\theta),y(\theta),C;\theta)\psi(\theta)-\gamma(1-\omega)\psi(\theta)$$

$$+\eta(\theta)e_{c_lc_l}(W(\theta),c_l(\theta),y(\theta),C;\theta)=0 \tag{51}$$

$C:$

$$-\lambda\int_\Theta e_C(W(\theta),c_l(\theta),y(\theta),C;\theta)f(\theta)d\theta+\gamma-\gamma\omega\int_\Theta e_C(W(\theta),c_l(\theta),y(\theta),C;\theta)\psi(\theta)d\theta=0 \tag{52}$$

$\kappa:$

$$\int_\Theta\eta(\theta)d\theta=0 \tag{53}$$

together with the boundary conditions $\mu(\underline{\theta})=\mu(\bar{\theta})=0$ and where $\epsilon(\theta)\equiv\frac{v'(\frac{y(\theta)}{\theta})}{v''(\frac{y(\theta)}{\theta})\frac{y(\theta)}{\theta}}$. Moreover, by implicit differentiation of $W(\theta)$ it follows that $e_W(W(\theta),c_l(\theta),y(\theta),C;\theta)=\frac{1}{u_{cn}(c_n(\theta),c_l(\theta))}$, $e_y(W(\theta),c_l(\theta),y(\theta),C;\theta)=\frac{v'\left(\frac{y(\theta)}{\theta}\right)\frac{1}{\theta}}{u_{cn}(c_n(\theta),c_l(\theta))}$, $e_{c_l}(W(\theta),c_l(\theta),y(\theta),C;\theta)=-\frac{u_{c_l}(c_n(\theta),c_l(\theta))}{u_{cn}(c_n(\theta),c_l(\theta))}$ and $e_C(W(\theta),c_l(\theta),y(\theta),C;\theta)=\frac{\alpha}{u_{cn}(c_n(\theta),c_l(\theta))}$. Moreover, observe that $e_{c_lc_l}(W(\theta),c_l(\theta),y(\theta),C;\theta)=\left[\frac{u_{c_l}(\theta)}{u_{cn}(\theta)}\frac{u_{cnc_l}(\theta)}{u_{cn}(\theta)}-\frac{u_{c_lc_l}(\theta)}{u_{cn(\theta)}}\right]$.

The result follows after manipulating (49)-(53). ∎

A.3 Proof of Proposition 3

Proof. Taking first order conditions in agent's problem we have

$$\frac{T'(y(\theta))}{1-T'(y(\theta))} = \frac{u_{c_n}(c_n^{eq}(\theta), c_l^{eq}(\theta))}{v'\left(\frac{y^{eq}(\theta)}{\theta}\right)\frac{1}{\theta}} - 1 \quad \forall \theta \in \Theta \tag{54}$$

and

$$\frac{u_{c_l}(c_n^{eq}(\theta), c_l^{eq}(\theta))}{u_{c_n}(c_n^{eq}(\theta), c_l^{eq}(\theta))} = 1 + \tau \quad \forall \theta \in \Theta \tag{55}$$

hence from (29)-(31) it follows that

$$\frac{u_{c_l}(c_n^{eq}(\theta), c_l^{eq}(\theta))}{u_{c_n}(c_n^{eq}(\theta), c_l^{eq}(\theta))} - 1 = \frac{\frac{\alpha(1-2\omega)}{\lambda}\int_\Theta \frac{\psi(\theta)}{B^*(\theta)}d\theta}{\int_\Theta \frac{f(\theta)}{B^*(\theta)}d\theta + \frac{\alpha\omega}{\lambda}\int_\Theta \frac{\psi(\theta)}{B^*(\theta)}d\theta} \quad \forall \theta \in \Theta \tag{56}$$

and

$$\frac{u_{c_n}(c_n^{eq}(\theta), c_l^{eq}(\theta))}{v'\left(\frac{y^{eq}(\theta)}{\theta}\right)\frac{1}{\theta}} - 1 = \frac{\alpha}{\lambda}\frac{\omega\psi(\theta)}{f(\theta)} + \frac{u_{c_n}(c_n^*(\theta), c_l^*(\theta))}{\theta f(\theta)}\left[1 + \frac{1}{\epsilon^*(\theta)}\right] I^*(\theta) \quad \forall \theta \in \Theta \tag{57}$$

Finally notice that the fact that the government balances its budget implies that

$$\int_\Theta c_n^{eq}(\theta) f(\theta) d\theta + \int_\Theta c_l^{eq}(\theta) f(\theta) d\theta = \int_\Theta y^{eq}(\theta) f(\theta) d\theta \tag{58}$$

Thus, from (56)-(58) we conclude that $\{c_n^{eq}(\theta), c_l^{eq}(\theta), y^{eq}(\theta)\}_{\theta\in\Theta} = \{c_n^*(\theta), c_l^*(\theta), y^*(\theta)\}_{\theta\in\Theta}$. ∎

B Solving the Model Numerically

I solve the model by casting it into a system of differential-algebraic equations. Let

$$r(\theta) \equiv \left(1 + \frac{1}{\epsilon(\theta)}\right)^{-1}\left[\frac{1}{v'(\frac{y(\theta)}{\theta})} - \frac{e_y(\theta)}{v'(\frac{y(\theta)}{\theta})}\left(1 + \frac{\gamma\omega}{\lambda}\frac{\psi(\theta)}{f(\theta)}\right)\right] \tag{59}$$

thus equation (50) becomes

$$r(\theta) = \frac{\mu(\theta)}{\lambda\theta^2 f(\theta)} \tag{60}$$

Differentiating (60) and using (59) we obtain

$$r'(\theta) = \frac{\mu'(\theta)}{\lambda f(\theta)\theta^2} - r(\theta)\left[\frac{2}{\theta} + \frac{f'(\theta)}{f(\theta)}\right] \tag{61}$$

Finally, using equation (49) we have

$$r'(\theta) = \frac{g(\theta)}{f(\theta)\lambda\theta^2} - \frac{e_W(\theta)}{\theta^2} - \frac{\gamma\omega}{\lambda}\frac{\psi(\theta)}{f(\theta)}\frac{e_W(\theta)}{\theta^2} - r(\theta)\left[\frac{2}{\theta} + \frac{f'(\theta)}{f(\theta)}\right] \tag{62}$$

From incentive compatibility we directly have

$$W'(\theta) = v'\left(\frac{y(\theta)}{\theta}\right)\frac{y(\theta)}{\theta^2}. \tag{63}$$

Thus, equations (62) and (63) are the differential equations of the system. The algebraic equations of the system are the following: from equation (51) we obtain

$$-e_{c_l}(\theta) - 1 - \frac{\gamma\omega\psi(\theta)e_{c_l}(\theta)}{\lambda f(\theta)} - \frac{\gamma(1-\omega)\psi(\theta)}{\lambda f(\theta)} + \eta(\theta)\frac{e_{c_l c_l}(\theta)}{\lambda f(\theta)} = 0 \tag{64}$$

from problem's restriction we have

$$e_{c_l}(\theta) - \kappa = 0. \tag{65}$$

Thus, equations (59), (64) and (65) are the algebraic equations of the system. The variables of the system are $[W(\theta), r(\theta), y(\theta), c_l(\theta), \eta(\theta)]'$. Once the value of these variables is known, it is straightforward to calculate the value of $c_n(\theta)$. The solution of the system has to satisfy the feasibility constraint and $\int_\Theta \eta(\theta)d\theta = 0$. This can be done by adjusting the value of κ and the vector of initial conditions.[13] Moreover, observe that an initial guess must be followed to calculate C, $\int_\theta c_n(\theta)\psi(\theta)d\theta$ and $\int_\theta c_n(\theta)f(\theta)d\theta$ which are required to obtain the solution of the system through the Lagrange multipliers of the system. We can iterate the previously guessed values until convergence.

[13]From the fact that $\mu(\bar{\theta})$ it must be that $r(\bar{\theta}) = 0$ whenever $\omega = 0$.

C Robusteness

In this section I present a sensitivity analysis of some of the parameters not presented in the main body of the paper. Figure 8 shows comparative statics of changes in the jealousy parameter, α when the skills distribution is Pareto. As expected, both taxes are increasing in this parameter. Also, observe that when α is higher, the luxury good consumption distribution experience sharper changes at the top upon the introduction of the positional flat tax. Figure 9 show changes in η, the parameter that measures the share of disposable income assigned ti each good. Figure 10 the endogenous distributions and taxes for different elasticity of substitution across goods, the parameter σ. Figure 11 display distributions and taxes for changes in the ratio $\psi(\theta)/f(\theta)$. Figure 12 also display changes in the ratio $\psi(\theta)/f(\theta)$. Notice that in this case, the previous ratio is decreasing in income! Finally, Figure 13 shows the endogenous distributions and taxes for a very high elasticity of labor supply.

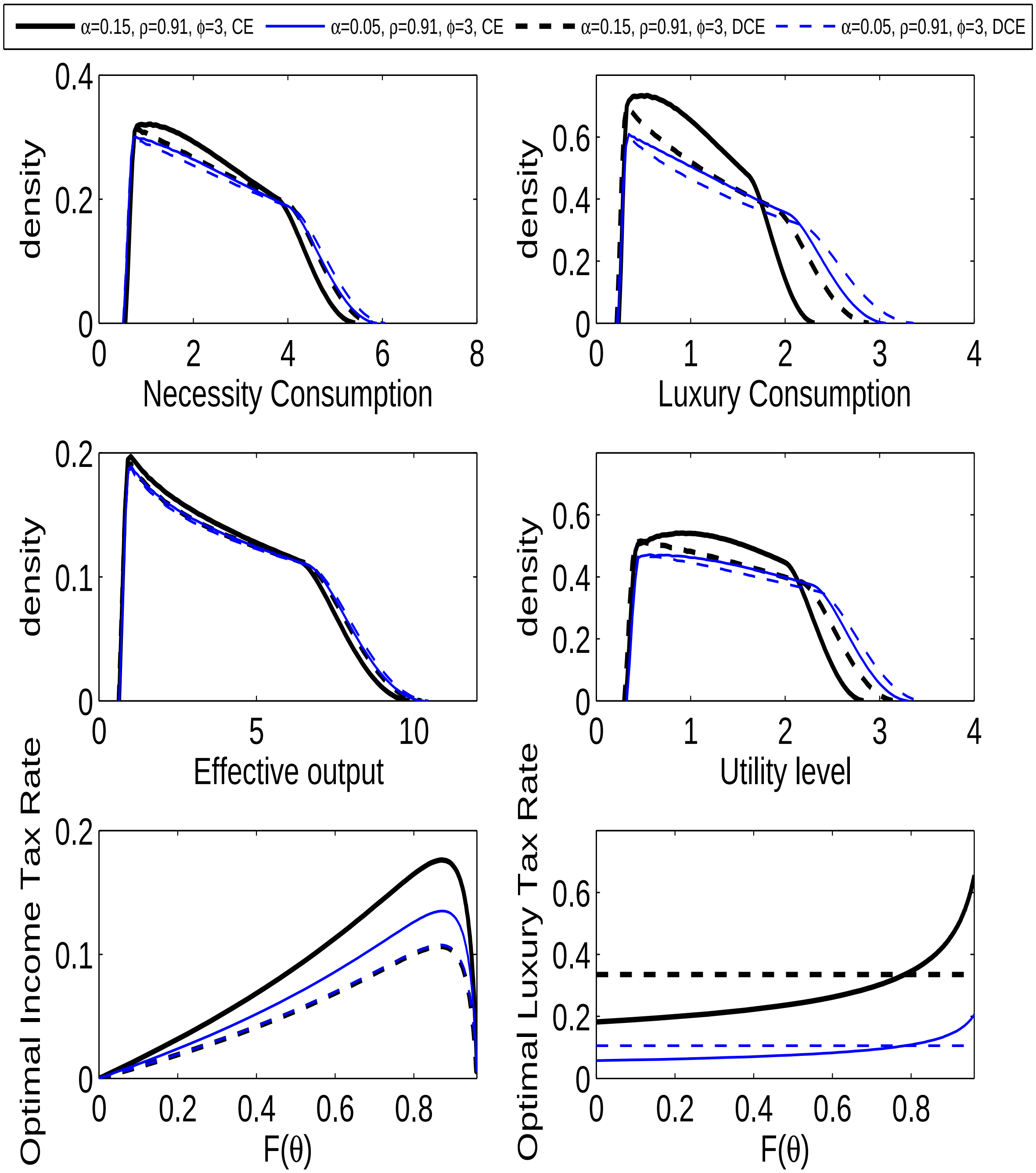

FIGURE 8: Endogenous distributions and optimal taxes when distribution of skills is Pareto and preferences are non-homothetic. Changes in α.

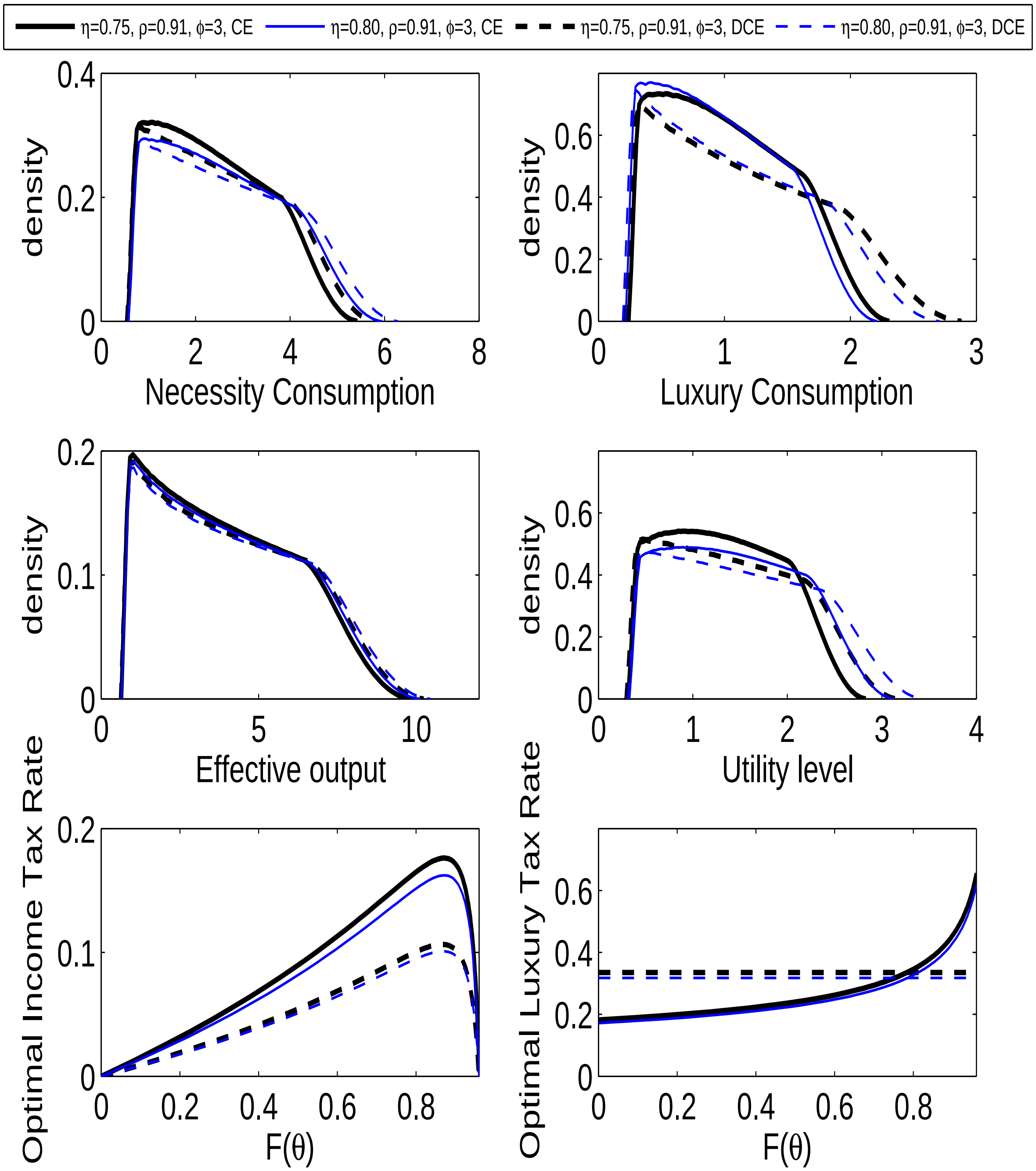

FIGURE 9: Endogenous distributions and optimal taxes when distribution of skills is Pareto and preferences are non-homothetic. Changes in η. When $\eta = 0.80$ we have $\lambda_\Theta = 0.15\%$, $\lambda_L = 1.21\%$ and $\lambda_H = -5\%$

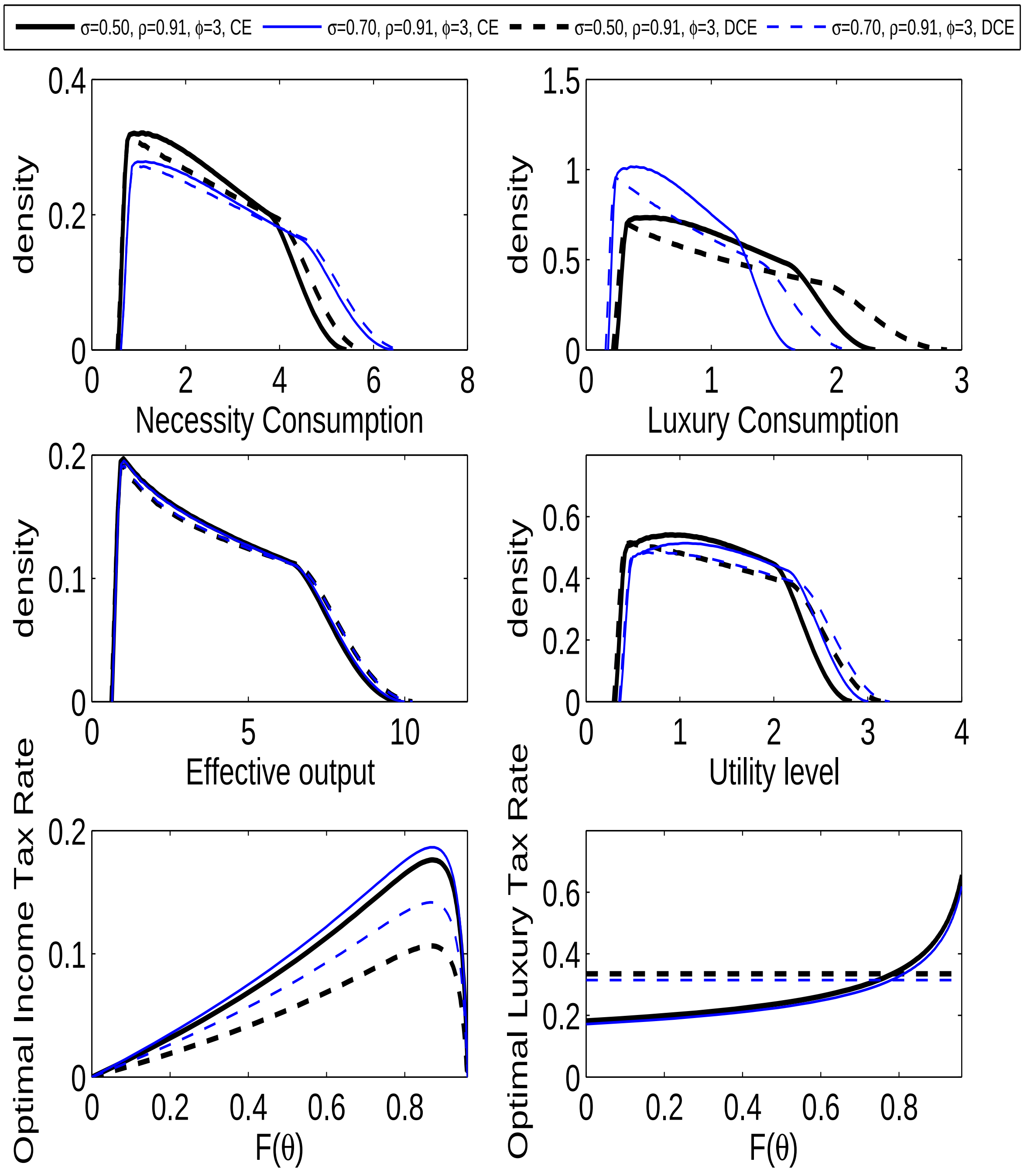

FIGURE 10: Endogenous distributions and optimal taxes when distribution of skills is Pareto and preferences are non-homothetic. Changes in σ. When $\sigma = 0.70$ we have $\lambda_\Theta = 0.10\%$, $\lambda_L = 0.87\%$ and $\lambda_H = -3.93\%$

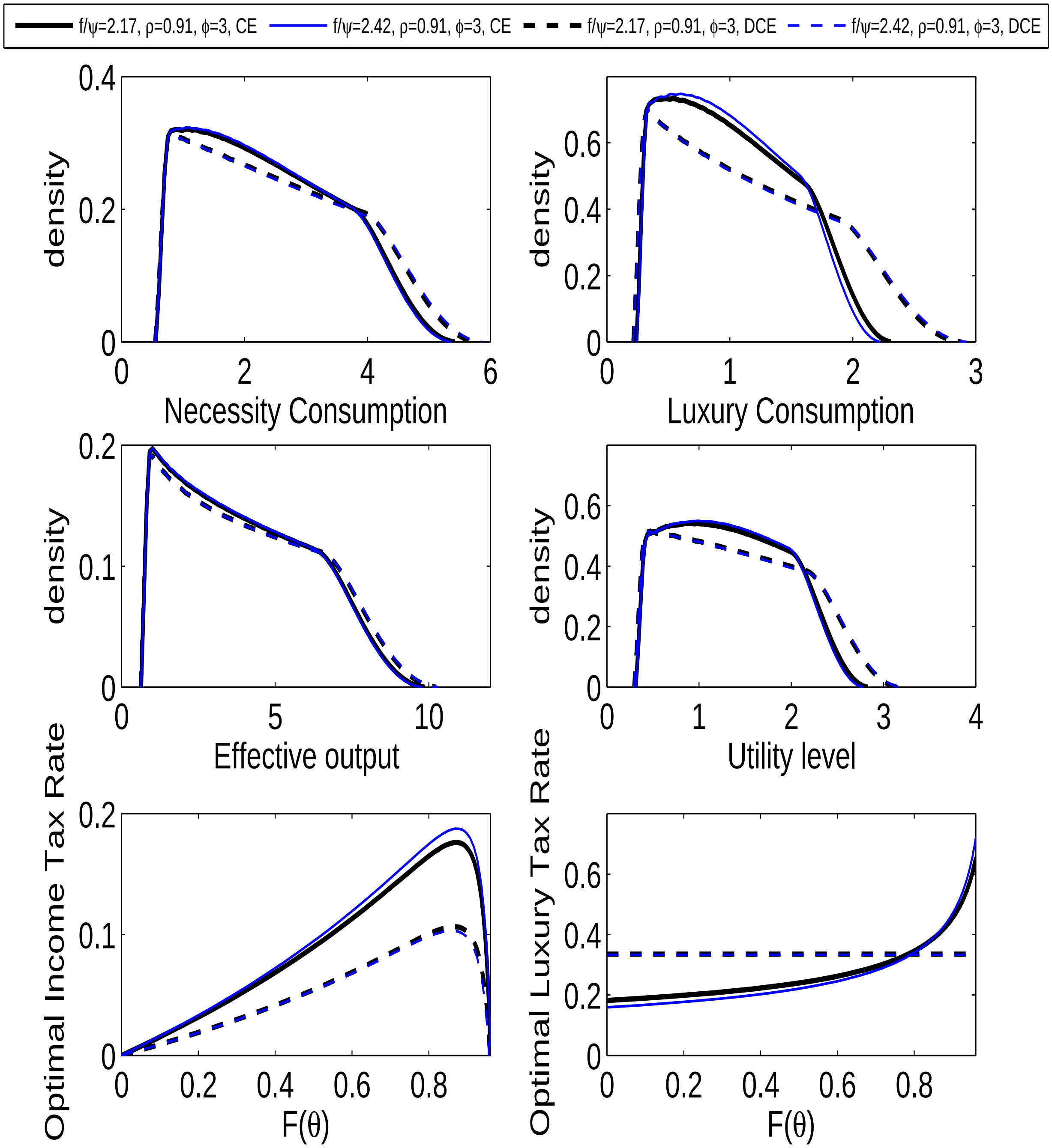

FIGURE 11: Endogenous distributions and optimal taxes when distribution of skills is Pareto and preferences are non-homothetic. Changes in k_ψ. When at the top, $\psi/f = 2.42$ we have $\lambda_\Theta = 0.21\%$, $\lambda_L = 1.62\%$ and $\lambda_H = -6.77\%$

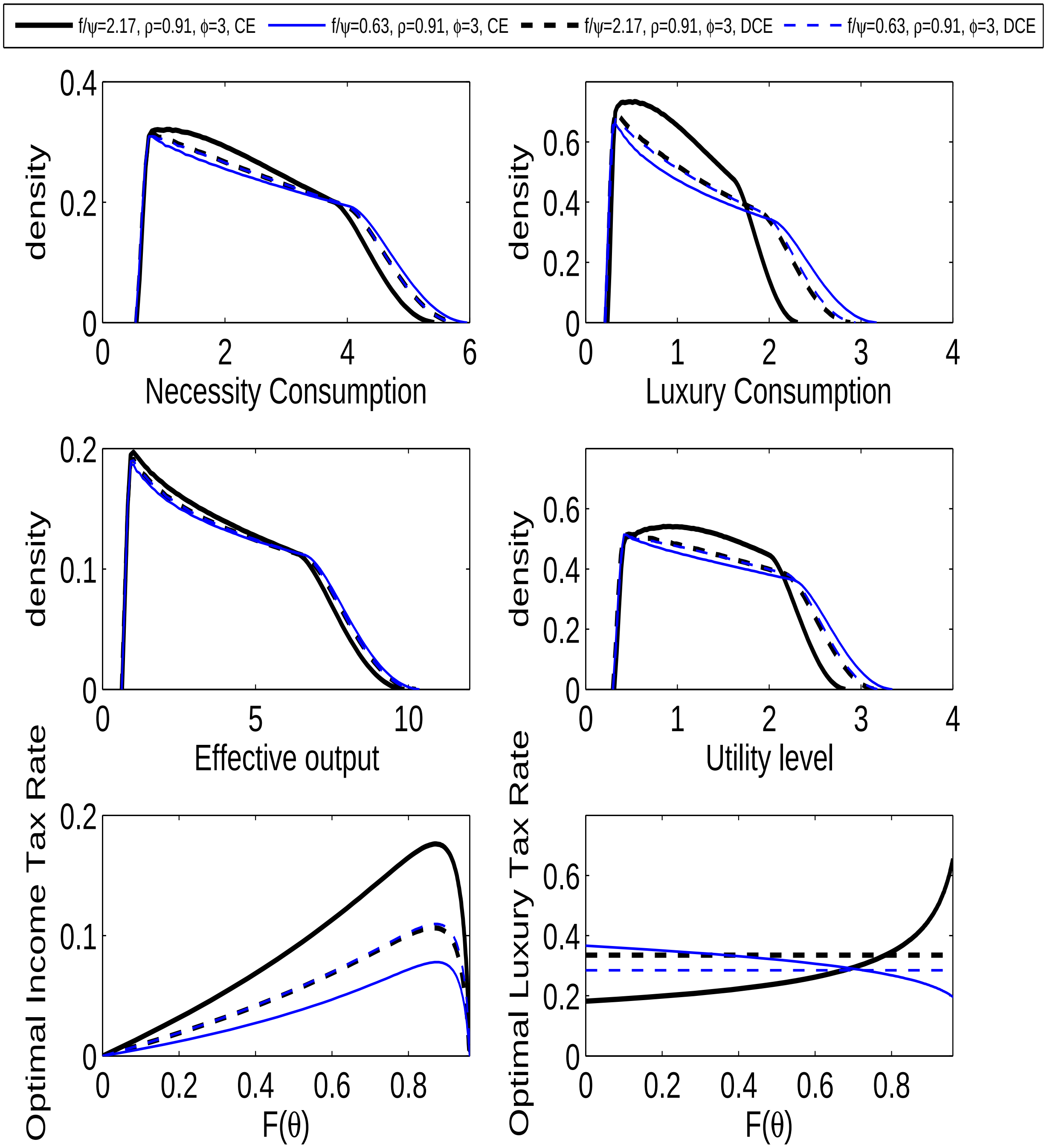

FIGURE 12: Endogenous distributions and optimal taxes when distribution of skills is Pareto and preferences are non-homothetic. Changes in k_ψ. When at the top, $\psi/f = 0.63$ and at the bottom $\psi/f = 1.20$ we have $\lambda_\Theta = 0.03\%$, $\lambda_L = -0.47\%$ and $\lambda_H = 2.51\%$

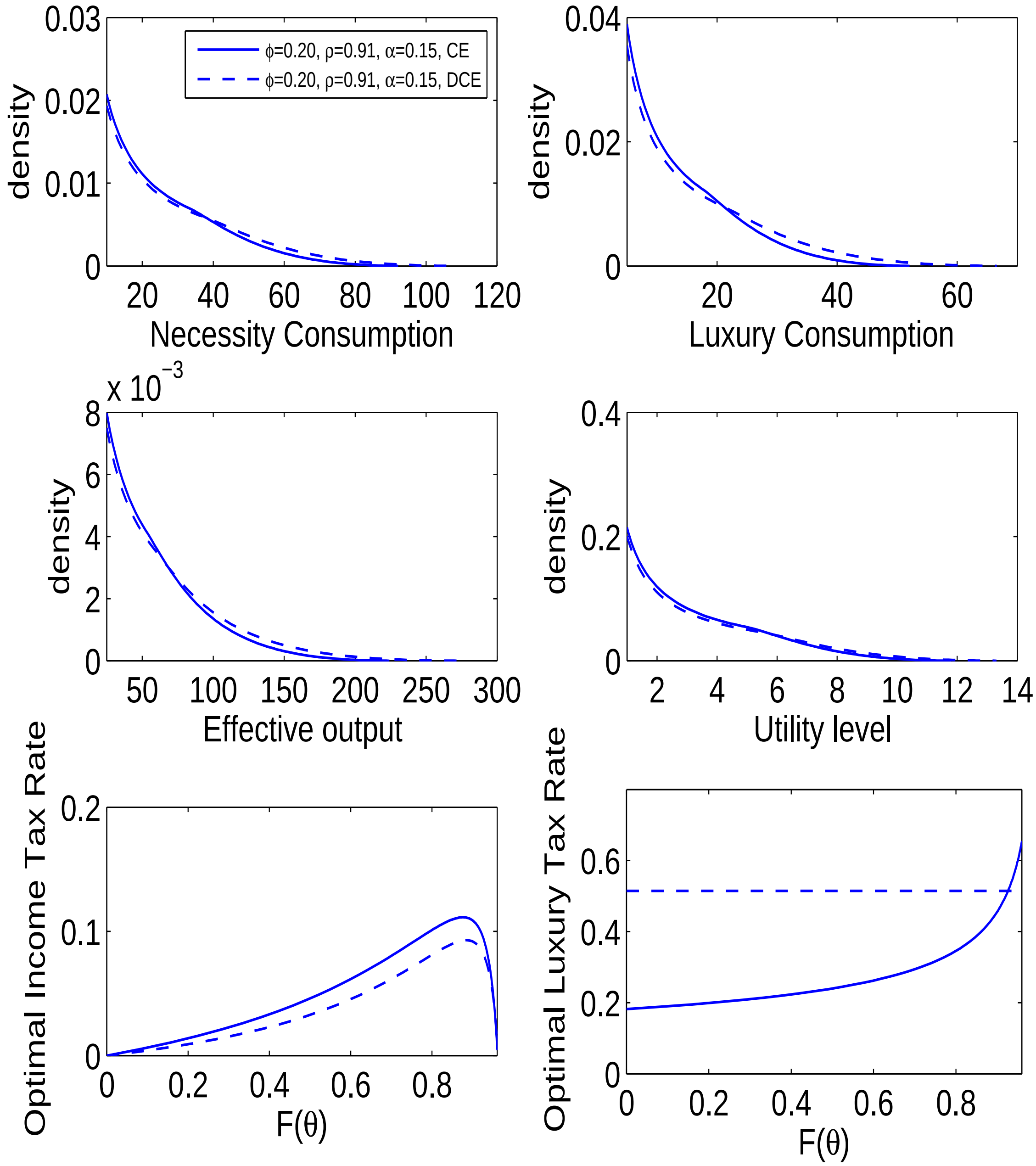

FIGURE 13: Upper tail of endogenous distributions and optimal taxes when distribution of skills is Pareto and preferences are non-homothetic. A very elastic labor supply ($\phi = 0.20$). We obtained $\lambda_\Theta = 0.32\%$, $\lambda_L = 1.85\%$ and $\lambda_H = -0.93\%$